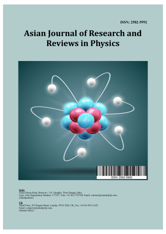

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Explicit Quasi-Rational Solutions and Parameter-Dependent Patterns for the Fifth Equation of the NLS Hierarchy

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

This study is part of a research program of rational solutions of the hierarchy of the nonlinear Schrödinger equation.
Here, we are interested in the equation of order 5 and we construct explicitly the first orders of rogue waves which were not yet found.
In particular, quasi rational solutions to the fifth equation of the NLS hierarchy are constructed. We give explicit expressions of these solutions for the first orders depending on multi-parameters. We study the patterns of these solutions in the (x, t) plane according to the different values of the parameters.

Keywords: Equation of order of the NLS hierarchy; rational solutions; rogue waves.

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1 INTRODUCTION

The fifth equation of the NLS hierarchy of order 5 $(NLS5)$ can be written as

$$\begin{array}{l} iu_t + u_{6x} + 12|u|^2u_{4x} + 2u^2\overline{u}_{4x} + 30u_{3x}u_x\overline{u} + 18u_{3x}u\overline{u}_x + 8u_xu\overline{u}_{3x} \\ +50u_{2x}|u_x|^2 + 50u_{2x}|u|^4 + 20u_{2x}^2\overline{u} + 22|u_{2x}|^2u + 20u_x^2\overline{u}_{2x} + 20|u|^2u^2\overline{u}_{2x}, \\ +10u^3\overline{u}_x^2 + 70u_x^2|u|^2\overline{u} + 60|u|^2|u_x|^2u + 20|u|^6u \end{array} \quad (1)$$

with as usual the subscript meaning the partial derivatives and $\overline{u}$ the complex conjugate of u.

This equation (1) is part of the hierarchy of NLS equations, as the NLS equation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], the first equation of this hierarchy, the mKdV equation [11, 12, 13, 14, 15] which is the second one, the LPD equation [16, 19, 20, 21, 22] which is the third one.

Here, explicit rational solutions for the first orders are constructed and the patterns of the modulus of the solutions in the (x, t) plane are studied.

2 QUASI RATIONAL SOLUTIONS TO THE NLS5 EQUATION

2.1 Quasi Rational Solutions of Order 1

Theorem 2.1. *The function $v(x,t)$ defined by*

$$v(x,t) == -\frac{\left(3 - 4\,x^2 - 14400\,t^2 + 480\,it\right)\mathrm{e}^{20\,it}}{1 + 4\,x^2 + 14400\,t^2} \quad (2)$$

is a solution to the $(NLS5)$ equation (1)

$$\begin{array}{l} iu_t + u_{6x} + 12|u|^2u_{4x} + 2u^2\overline{u}_{4x} + 30u_{3x}u_x\overline{u} + 18u_{3x}u\overline{u}_x + 8u_xu\overline{u}_{3x} \\ +50u_{2x}|u_x|^2 + 50u_{2x}|u|^4 + 20u_{2x}^2\overline{u} + 22|u_{2x}|^2u + 20u_x^2\overline{u_{2x}} + 20|u|^2u^2\overline{u}_{2x}, \\ +10u^3\overline{u}_x^2 + 70u_x^2|u|^2\overline{u} + 60|u|^2|u_x|^2u + 20|u|^6u. \end{array}$$

Proof: It is sufficient to replace the expression of the solution given by (2) and check that (1) is verified.

The solution of order 1 is represented in Fig. 1.

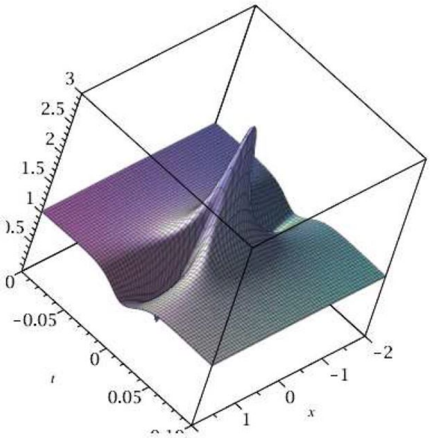

Fig. 1. Solution of order 1 to $(NLS5)$

We get a smooth solution of the equation (1).

2.2 Quasi Rational Solutions of Order 2 Depending on 2 Real Parameters

Theorem 2.2. *The function $v(x,t)$ defined by*

$$v(x,t) = \frac{n(x,t)}{d(x,t)} \tag{3}$$

with

$$n(x,t) = -(-64\,x^6 + 2304\,b_1x^5 + 768\,ia_1x^4 - 34560\,{b_1}^2x^4 - 691200\,t^2x^4 + 23040\,itx^4 - 46080\,a_1tx^4 - 768\,{a_1}^2x^4 + 144\,x^4 - 18432\,ia_1x^3b_1 -$$
$$552960\,itx^3b_1 + 16588800\,b_1t^2x^3 + 1105920\,b_1a_1tx^3 + 18432\,b_1{a_1}^2x^3 - 4992\,b_1x^3 + 276480\,{b_1}^3x^3 + 5760\,{a_1}^2x^2 - 9953280\,{b_1}^2a_1tx^2 + 180\,x^2 +$$
$$165888\,ia_1{b_1}^2x^2 - 16588800\,{a_1}^2t^2x^2 + 552960\,i{a_1}^2tx^2 - 165888\,{b_1}^2{a_1}^2x^2 - 368640\,{a_1}^3tx^2 + 529920\,a_1tx^2 - 1152\,ia_1x^2 - 331776000\,a_1t^3x^2 +$$
$$4976640\,it{b_1}^2x^2 + 16588800\,ia_1t^2x^2 + 165888000\,it^3x^2 + 10713600\,t^2x^2 - 2488320000\,t^4x^2 - 1244160\,{b_1}^4x^2 + 58752\,{b_1}^2x^2 - 3072\,{a_1}^4x^2 -$$
$$126720\,itx^2 - 149299200\,{b_1}^2t^2x^2 + 6144\,i{a_1}^3x^2 - 19906560\,itx{b_1}^3 + 36864\,b_1{a_1}^4x - 111974400\,b_1t^2x + 4423680\,b_1{a_1}^3tx - 290304\,{b_1}^3x +$$
$$39813120\,{b_1}^3a_1tx + 3981312000\,b_1a_1t^3x + 967680\,itb_1x - 73728\,i{a_1}^3xb_1 - 663552\,ia_1x{b_1}^3 - 4608\,ia_1xb_1 - 6635520\,i{a_1}^2txb_1 - 199065600\,ia_1t^2xb_1 -$$
$$1990656000\,it^3xb_1 - 50688\,b_1{a_1}^2x + 597196800\,{b_1}^3t^2x - 5616\,b_1x + 2985984\,{b_1}^5x + 199065600\,b_1{a_1}^2t^2x + 663552\,{b_1}^3{a_1}^2x - 5253120\,b_1a_1tx +$$
$$29859840000\,b_1t^4x + 207360000\,t^4 - 619200\,t^2 + 597196800\,ia_1t^2{b_1}^2 - 55296000\,{a_1}^4t^2 - 2211840000\,{a_1}^3t^3 + 19906560\,i{a_1}^2t{b_1}^2 + 23500800\,{a_1}^2t^2 +$$
$$248832000\,a_1t^3 - 597196800000\,a_1t^5 + 1536\,i{a_1}^3 - 2985984000000\,t^6 + 298598400000\,it^5 - 110592\,{b_1}^2{a_1}^4 + 96768\,{b_1}^2{a_1}^2 + 158400\,a_1t +$$
$$12288\,i{a_1}^5 + 373248000\,it^3 - 44640\,it - 45 - 995328\,{b_1}^4{a_1}^2 - 895795200\,{b_1}^4t^2 - 597196800\,{b_1}^2{a_1}^2t^2 - 49766400000\,{a_1}^2t^4 - 737280\,{a_1}^5t +$$
$$286156800\,{b_1}^2t^2 + 995328\,ia_1{b_1}^4 + 29859840\,it{b_1}^4 + 506880\,i{a_1}^2t + 26265600\,ia_1t^2 - 720\,ia_1 + 221184\,i{a_1}^3{b_1}^2 + 5971968000\,it^3{b_1}^2 +$$
$$768000\,{a_1}^3t - 13271040\,{b_1}^2{a_1}^3t - 1244160\,it{b_1}^2 - 11943936000\,{b_1}^2a_1t^3 - 59719680\,{b_1}^4a_1t + 3317760000\,i{a_1}^2t^3 + 49766400000\,ia_1t^4 +$$
$$518400\,{b_1}^4 + 8448\,{a_1}^4 - 2985984\,{b_1}^6 - 4096\,{a_1}^6 + 1872\,{a_1}^2 + 18000\,{b_1}^2 - 89579520000\,{b_1}^2t^4 + 69120\,ia_1{b_1}^2 + 12441600\,{b_1}^2a_1t + 1843200\,i{a_1}^4t +$$
$$110592000\,i{a_1}^3t^2)\mathrm{e}^{2\,i(a_1+10\,t)}$$

and

$$d(x,t) = 64\,x^6 - 2304\,b_1x^5 + 46080\,a_1tx^4 + 48\,x^4 + 691200\,t^2x^4 + 768\,{a_1}^2x^4 + 34560\,{b_1}^2x^4 - 18432\,b_1{a_1}^2x^3 - 1105920\,b_1a_1tx^3 + 384\,b_1x^3 -$$
$$16588800\,b_1t^2x^3 - 276480\,{b_1}^3x^3 + 2488320000\,t^4x^2 + 149299200\,{b_1}^2t^2x^2 + 331776000\,a_1t^3x^2 - 6566400\,t^2x^2 + 9953280\,{b_1}^2a_1tx^2 - 253440\,a_1tx^2 +$$
$$108\,x^2 + 1244160\,{b_1}^4x^2 - 17280\,{b_1}^2x^2 - 1152\,{a_1}^2x^2 + 368640\,{a_1}^3tx^2 + 16588800\,{a_1}^2t^2x^2 + 3072\,{a_1}^4x^2 + 165888\,{b_1}^2{a_1}^2x^2 + 1935360\,b_1a_1tx -$$
$$4423680\,b_1{a_1}^3tx - 199065600\,b_1{a_1}^2t^2x + 124416\,{b_1}^3x - 663552\,{b_1}^3{a_1}^2x - 2448\,b_1x + 62208000\,b_1t^2x - 39813120\,{b_1}^3a_1tx - 2985984\,{b_1}^5x -$$
$$29859840000\,b_1t^4x - 4608\,b_1{a_1}^2x - 3981312000\,b_1a_1t^3x - 36864\,b_1{a_1}^4x - 597196800\,{b_1}^3t^2x + 59443200\,{a_1}^2t^2 + 1075200\,{a_1}^3t - 136857600\,{b_1}^2t^2 +$$
$$1410048000\,a_1t^3 + 69120\,{b_1}^2{a_1}^2 + 2985984000000\,t^6 + 9259200\,t^2 - 2488320\,{b_1}^2a_1t + 89579520000\,{b_1}^2t^4 + 995328\,{b_1}^4{a_1}^2 + 9 + 110592\,{b_1}^2{a_1}^4 +$$
$$49766400000\,{a_1}^2t^4 + 737280\,{a_1}^5t + 895795200\,{b_1}^4t^2 + 2211840000\,{a_1}^3t^3 + 597196800000\,a_1t^5 + 233280\,a_1t + 55296000\,{a_1}^4t^2 + 12234240000\,t^4 +$$

$13271040\, b_1{}^2 a_1{}^3 t + 597196800\, b_1{}^2 a_1{}^2 t^2 + 11943936000\, b_1{}^2 a_1 t^3 + 59719680\, b_1{}^4 a_1 t - 269568\, b_1{}^4 + 6912\, a_1{}^4 + 2985984\, b_1{}^6 + 4096\, a_1{}^6 + 1584\, a_1{}^2 + 20016\, b_1{}^2$

is a solution to the (NLS5) equation (1).

Proof: Replacing the expression of the solution given by (3), we check that the relation (1) is verified.

Solutions of order 2 are represented in Figs. 2 and 3.

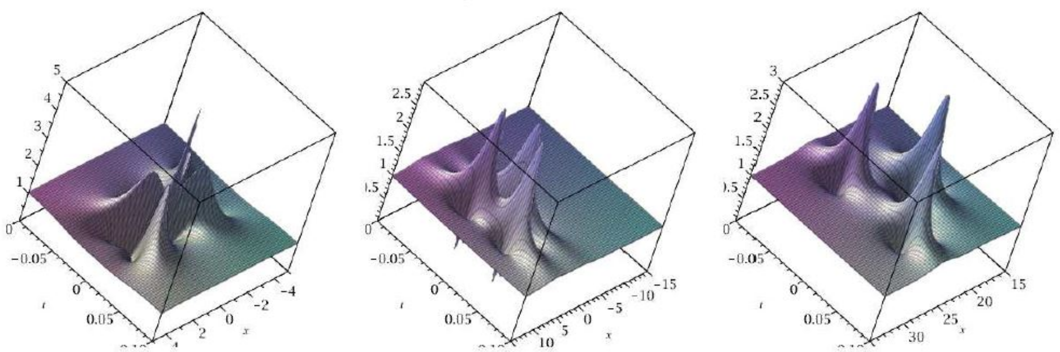

Fig. 2. Solution of order 2 to the equation (1); to the left $a_1 = 0$, $b_1 = 0$; in the center $a_1 = 0$, $b_1 = 1$; to the right $a_1 = 0$, $b_1 = 4$.

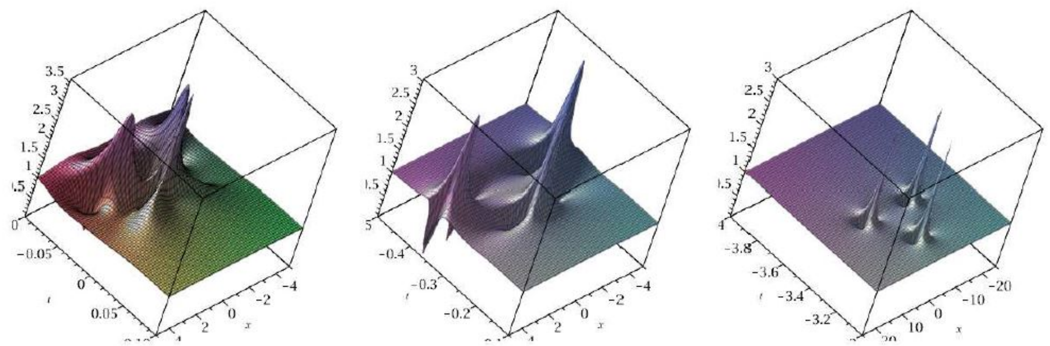

Fig. 3. Solution of order 2 to the equation (1); to the left $a_1 = 1$, $b_1 = 0$; in the center $a_1 = 10$, $b_1 = 1$; to the right $a_1 = 100$, $b_1 = 100$.

When one or both parameters increase, three peaks appear. When only one of the parameters increases, the three peaks appear but with different orientations.

2.3 Quasi Rational Solutions of Order 3 Depending on 4 Real Parameters

The solution depending on 4 real parameters being too long, we only present in the appendix. Here we give the solution without parameters.

Theorem 2.3. *The function* $v(x,t)$ *defined by*

$$v(x,t) = \frac{n(x,t)}{d(x,t)} \tag{4}$$

with

$$n(x,t) = -(-4096\,x^{12} + 2949120\,itx^{10} + 18432\,x^{10} - 88473600\,t^2x^{10} + 57600\,x^8 - 40550400\,ix^8t + 3428352000\,t^2x^8 + 53084160000\,it^3x^8 - 796262400000\,t^4x^8 + 90316800\,itx^6 - 1220935680000\,ix^6t^3 - 34854912000\,t^2x^6 - 3822059520000000\,t^6x^6 + 172800\,x^6 + 382205952000000\,it^5x^6 + 35300966400000\,t^4x^6 - 1285632000\,t^2x^4 - 5828640768000000\,ix^4t^5 + 137594142720000000\,it^7x^4 - 226800\,x^4 + 37125734400000\,t^4x^4 + 123261419520000000\,t^6x^4 + 37324800\,itx^4 - 4651499520000\,ix^4t^3 - 103195607040000000000\,t^8x^4 - 229970534400000\,t^4x^2 - 206391214080000000\,ix^2t^7 - 131888217600\,t^2x^2 + 485740800\,itx^2 + 139314069504000000000\,t^8x^2 - 113400\,x^2 + 11588935680000\,it^3x^2 - 148601674137600000000\,t^{10}x^2 + 66002190336000000\,it^5x^2 + 247669456896000000000\,it^9x^2 - 1059044917248000000\,t^6x^2 + 58190400\,it - 61123092480000000\,it^5 - 891610044825600000000000\,t^{12} + 19761958748160000000\,it^7 + 14175 + 17832200896512000000000\,it^{11} - 17828771328000\,it^3 + 179560356249600000000\,it^9 + 62368963200\,t^2 + 72997992529920000000\,t^8 - 645625935360000\,t^4 + 2630186090496000000\,t^6 - 1362182012928000000000\,t^{10})e^{20\,it}$$

and

$$d(x,t) = 4096\,x^{12} + 6144\,x^{10} + 88473600\,t^2x^{10} - 2101248000\,t^2x^8 + 796262400000\,t^4x^8 + 34560\,x^8 + 19372032000\,t^2x^6 + 149760\,x^6 + 3822059520000000\,t^6x^6 - 19375718400000\,t^4x^6 + 10319560704000000000\,t^8x^4 + 54000\,x^4 - 429981696000000\,t^6x^4 - 51079680000\,t^2x^4 - 176471654400000\,t^4x^4 + 1663840051200000\,t^4x^2 + 46438023168000000000\,t^8x^2 - 8867750400\,t^2x^2 + 148601674137600000000\,t^{10}x^2 + 1179439792128000000\,t^6x^2 + 48600\,x^2 + 2025 + 891610044825600000000000\,t^{12} + 51261206400\,t^2 + 771516157132800000000\,t^8 + 704698652160000\,t^4 - 4230900449280000000\,t^6 + 177083661680640000000000\,t^{10}$$

is a solution to the $(NLS5)$ *equation* (1).

Proof: It is sufficient to check that the relation (1) is verified when we replace the expression of the solution given by (5).

In the following, patterns of the modules of the solutions are studied according to different values of the parameters.

The solutions of order 3 depending on 4 real parameters are represented in Figs. $4-7$.

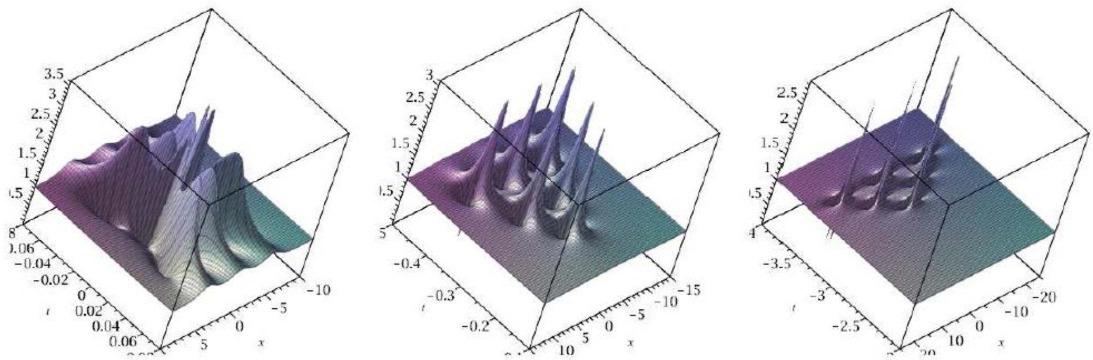

Fig. 4. Solution of order 3 **to** (1)**; to the left** $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$**; in the center** $a_1 = 10$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$**; to the right** $a_1 = 100$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0$**.**

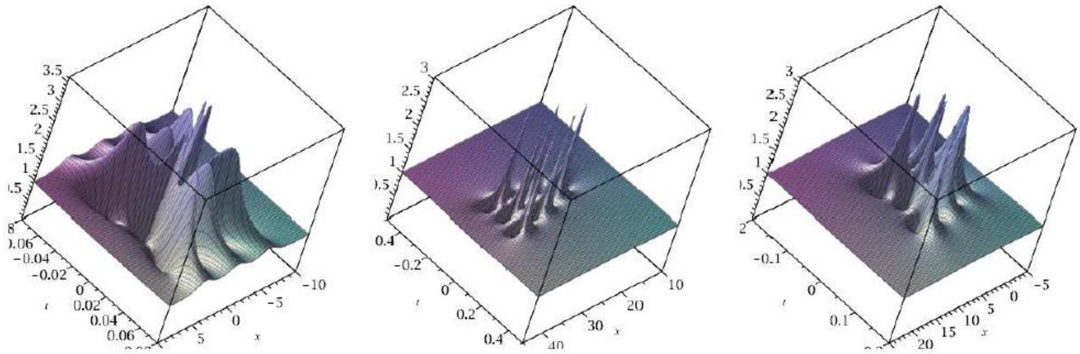

Fig. 5. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0, 1$, $a_2 = 0$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 0$; to the right $a_1 = 0$, $b_1 = 10$, $a_2 = 0$, $b_2 = 0$.

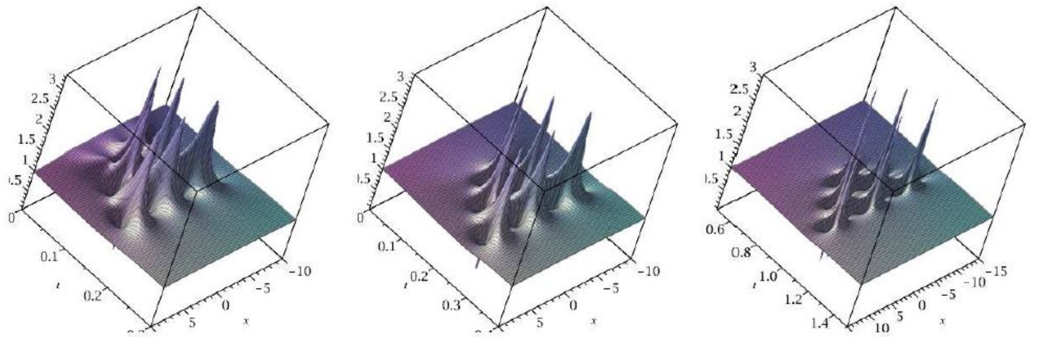

Fig. 6. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0, 5$, $b_2 = 0$; in the center $a_1 = 0$, $b_1 = 0$, $a_2 = 1$, $b_2 = 0$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 5$, $b_2 = 0$.

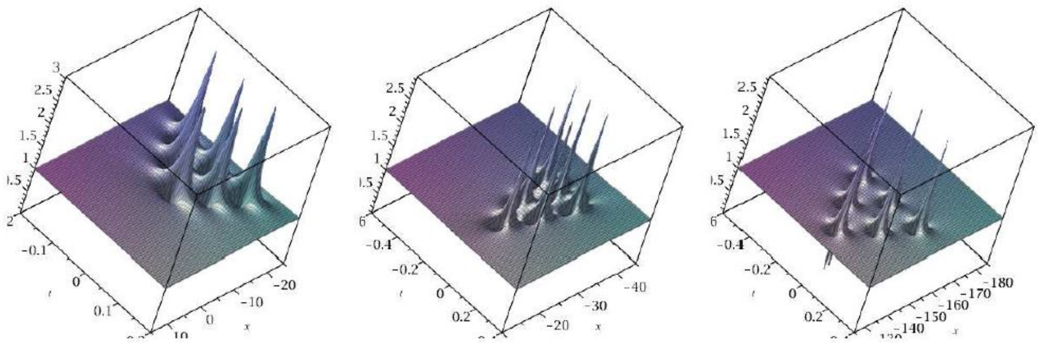

Fig. 7. Solution of order 3 to (1); to the left $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 0, 5$; in the center $a_1 = 0$, $b_1 = 0$, $a_2 = 0$, $b_2 = 1$; to the right $a_1 = 0$, $b_1 = 5$, $a_2 = 0$, $b_2 = 5$.

As other equations belonging to this NLS hierarchy, for example, the NLS equation [23], the mKdV equation [24], or the Lakshmanan Porsezian Daniel equation [25], we recover the structure of triangles with peaks which appear in function of the different values of the parameters.

3 CONCLUSION

This study is part of a research program of rational solutions of the hierarchy of the nonlinear Schrödinger equation. Here, the equation of order 5 is considered and the first orders of rogue waves have been explicitly constructed. To the best of my knowledge, these solutions were not yet found.

In particular, rational solutions to the $(NLS5)$ equation have been given for the first orders. In all these N-order solutions we get quotient of a polynomial of degree $N(N+1)$ in x and t for the numerator by a polynomial of degree $N(N+1)$ in x and t for the denominator.

In the case of solutions of order 2, the solutions depend on two real parameters, and the structure of triangles with three peaks is observed for their modules.

For the case of solutions of order 3, the solutions depend on four real parameters. In the plane (x,t) of the coordinates, the representation of the modules of the solutions reveals the formation of triangles containing 6 peaks.

DISCLAIMER (ARTIFICIAL INTELLIGENCE)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of manuscripts.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX

Solution of order 3 to the (NLS5) equation depending on 4 real parameters :
The function $v(x,t)$ defined by

$$v(x,t) = \left(1 - 24\frac{n(x,t)}{d(x,t)}\right) \mathrm{e}^{i(2\,a_1 - 6\,a_2 + 20\,t)} \tag{5}$$

with

$$n(x,t) = 675 + 353894400\,t^2 + 91800\,(16\,a_2 - 160\,t)^2 + 2190\,(4\,a_1 - 24\,a_2 + 120\,t)^6 + 495\,(4\,a_1 - 24\,a_2 + 120\,t)^8 + 11\,(4\,a_1 - 24\,a_2 + 120\,t)^{10} + 88473600\,{b_2}^2 + (2\,x - 12\,b_1 + 60\,b_2)^{10} + 27000\,(8\,b_1 - 80\,b_2)^2 - 11059200\,(16\,a_2 - 160\,t)t + i(1857600\,t + 64800\,(16\,a_2 - 160\,t)^3 - 870\,(4\,a_1 - 24\,a_2 + 120\,t)^7 + 25\,(4\,a_1 - 24\,a_2 + 120\,t)^9 + (4\,a_1 - 24\,a_2 + 120\,t)^{11} - 151200\,a_2 - 5529600\,(16\,a_2 - 160\,t)^2 t - 90\,(4\,a_1 - 24\,a_2 + 120\,t)^8 (16\,a_2 - 160\,t) - 120\,(4\,a_1 - 24\,a_2 + 120\,t)^6 (80\,a_2 - 1248\,t) + 900\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (464\,a_2 - 4000\,t) + 5529600\,(8\,b_1 - 80\,b_2)^2 t + (-240\,(4\,a_1 - 24\,a_2 + 120\,t)^7 (8\,b_1 - 80\,b_2) - 7200\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 10800\,(4\,a_1 - 24\,a_2 + 120\,t)(24\,b_1 - 400\,b_2 + 4\,(8\,b_1 - 80\,b_2)^3 + 4\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t)^2) + 3600\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (24\,b_1 - 176\,b_2) + 720\,(4\,a_1 - 24\,a_2 + 120\,t)^5 (56\,b_1 - 400\,b_2) + 21600\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 1382400\,(16\,a_2 - 160\,t)b_2 - 2764800\,(8\,b_1 - 80\,b_2)t - 43200\,(4\,a_1 - 24\,a_2 + 120\,t)^2 ((8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 32\,(16\,a_2 - 160\,t)b_2 - 64\,(8\,b_1 - 80\,b_2)t))(2\,x - 12\,b_1 + 60\,b_2) + 90\,(4\,a_1 - 24\,a_2 + 120\,t)^5 (-107 + 28\,(8\,b_1 - 80\,b_2)^2 + 12\,(16\,a_2 - 160\,t)^2) - 21600\,(8\,b_1 - 80\,b_2)^2 (16\,a_2 - 160\,t) + 5400\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (176\,a_2 - 2464\,t + 4\,(8\,b_1 - 80\,b_2)^2 (16\,a_2 - 160\,t) + 4\,(16\,a_2 - 160\,t)^3) - 225\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (11 + 80\,(8\,b_1 - 80\,b_2)^2 + 80\,(16\,a_2 - 160\,t)^2 + 4096\,(8\,b_1 - 80\,b_2)b_2 + 8192\,(16\,a_2 - 160\,t)t) - 675\,(4\,a_1 - 24\,a_2 + 120\,t)(-7 + 56\,(8\,b_1 - 80\,b_2)^2 + 88\,(16\,a_2 - 160\,t)^2 - 4096\,(8\,b_1 - 80\,b_2)b_2 - 131072\,{b_2}^2 - 524288\,t^2) + (4\,a_1 - 24\,a_2 + 120\,t)(2\,x - 12\,b_1 + 60\,b_2)^{10} + (-60\,a_1 + 840\,a_2 - 6600\,t + 5\,(4\,a_1 - 24\,a_2 + 120\,t)^3)(2\,x - 12\,b_1 + 60\,b_2)^8 + (-600\,a_1 - 240\,a_2 + 58800\,t - 140\,(4\,a_1 - 24\,a_2 + 120\,t)^3 + 10\,(4\,a_1 - 24\,a_2 + 120\,t)^5 + 240\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (16\,a_2 - 160\,t))(2\,x - 12\,b_1 + 60\,b_2)^6 + (-240\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (8\,b_1 - 80\,b_2) - 1440\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 720\,(4\,a_1 - 24\,a_2 + 120\,t)(8\,b_1 - 176\,b_2))(2\,x - 12\,b_1 + 60\,b_2)^5 + (-450\,(4\,a_1 - 24\,a_2 + 120\,t)^3 - 210\,(4\,a_1 - 24\,a_2 + 120\,t)^5 + 10\,(4\,a_1 - 24\,a_2 + 120\,t)^7 + 300\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (16\,a_2 - 160\,t) + 450\,(4\,a_1 - 24\,a_2 + 120\,t)(-3 + 12\,(8\,b_1 - 80\,b_2)^2 - 4\,(16\,a_2 - 160\,t)^2) - 14400\,a_2 + 259200\,t + 1800\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (16\,a_2 - 224\,t))(2\,x - 12\,b_1 + 60\,b_2)^4 + (-480\,(4\,a_1 - 24\,a_2 + 120\,t)^5 (8\,b_1 - 80\,b_2) + 14400\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 7200\,(4\,a_1 - 24\,a_2 + 120\,t)(8\,b_1 - 48\,b_2) - 2400\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (16\,b_1 - 128\,b_2) - 14400\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) - 460800\,(16\,a_2 - 160\,t)b_2 + 921600\,(8\,b_1 - 80\,b_2)t)(2\,x - 12\,b_1 + 60\,b_2)^3 + (1710\,(4\,a_1 - 24\,a_2 + 120\,t)^5 - 60\,(4\,a_1 - 24\,a_2 + 120\,t)^7 + 5\,(4\,a_1 - 24\,a_2 + 120\,t)^9 - 900\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (7 + 4\,(8\,b_1 - 80\,b_2)^2 - 12\,(16\,a_2 - 160\,t)^2) + 675\,(4\,a_1 - 24\,a_2 + 120\,t)(7 + 16\,(8\,b_1 - 80\,b_2)^2 + 16\,(16\,a_2 - 160\,t)^2) - 345600\,a_2 + 4492800\,t - 21600\,(8\,b_1 - 80\,b_2)^2 (16\,a_2 - 160\,t) - 21600\,(16\,a_2 - 160\,t)^3 + 691200\,(4\,a_1 - 24\,a_2 + 120\,t)^2 t - 1800\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (64\,a_2 - 448\,t))(2\,x - 12\,b_1 + 60\,b_2)^2 - 5529600\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t)b_2) + 15\,(1 + (4\,a_1 - 24\,a_2 + 120\,t)^2)(2\,x - 12\,b_1 + 60\,b_2)^8 + (210 - 60\,(4\,a_1 - 24\,a_2 + 120\,t)^2 + 50\,(4\,a_1 - 24\,a_2 + 120\,t)^4 + 480\,(4\,a_1 - 24\,a_2 + 120\,t)(16\,a_2 - 160\,t))(2\,x - 12\,b_1 + 60\,b_2)^6 + (-720\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (8\,b_1 - 80\,b_2) - 5760\,b_1 - 11520\,b_2)(2\,x - 12\,b_1 + 60\,b_2)^5 + (450\,(4\,a_1 - 24\,a_2 + 120\,t)^2 - 150\,(4\,a_1 - 24\,a_2 + 120\,t)^4 + 70\,(4\,a_1 - 24\,a_2 + 120\,t)^6 + 1200\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (16\,a_2 - 160\,t) - 450 + 5400\,(8\,b_1 - 80\,b_2)^2 - 1800\,(16\,a_2 - 160\,t)^2 + 3600\,(4\,a_1 - 24\,a_2 + 120\,t)(16\,a_2 - 224\,t))(2\,x - 12\,b_1 + 60\,b_2)^4 + (-2400\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (8\,b_1 - 80\,b_2) + 28800\,(4\,a_1 - 24\,a_2 + 120\,t)(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 57600\,b_1 - 806400\,b_2 - 7200\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (16\,b_1 - 128\,b_2))(2\,x - 12\,b_1 + 60\,b_2)^3 + (6750\,(4\,a_1 - 24\,a_2 + 120\,t)^4 + 420\,(4\,a_1 - 24\,a_2 + 120\,t)^6 + 45\,(4\,a_1 - 24\,a_2 + 120\,t)^8 - 2700\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (5 + 4\,(8\,b_1 - 80\,b_2)^2 - 12\,(16\,a_2 - 160\,t)^2) - 675 - 10800\,(8\,b_1 - 80\,b_2)^2 - 10800\,(16\,a_2 - 160\,t)^2 + 21600\,(4\,a_1 - 24\,a_2 + 120\,t)(32\,a_2 - 384\,t) - 7200\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (32\,a_2 - 128\,t))(2\,x - 12\,b_1 + 60\,b_2)^2 + (-1680\,(4\,a_1 - 24\,a_2 + 120\,t)^6 (8\,b_1 - 80\,b_2) - 28800\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) - 10800\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (8\,b_1 - 272\,b_2) + 86400\,b_1 - 1209600\,b_2 + 43200\,(8\,b_1 - 80\,b_2)^3 + 43200\,(8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t)^2 + 3600\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (8\,b_1 + 80\,b_2) - 86400\,(4\,a_1 - 24\,a_2 + 120\,t)((8\,b_1 - 80\,b_2)(16\,a_2 - 160\,t) + 32\,(16\,a_2 - 160\,t)b_2 - 64\,(8\,b_1 - 80\,b_2)t))(2\,x - 12\,b_1 + 60\,b_2) + 450\,(4\,a_1 - 24\,a_2 + 120\,t)^4 (-17 + 28\,(8\,b_1 - 80\,b_2)^2 + 12\,(16\,a_2 - 160\,t)^2) + 10800\,(4\,a_1 - 24\,a_2 + 120\,t)(-16\,a_2 + 224\,t + 4\,(8\,b_1 - 80\,b_2)^2 (16\,a_2 - 160\,t) + 4\,(16\,a_2 - 160\,t)^3) + 675\,(4\,a_1 - 24\,a_2 + 120\,t)^2 (-3 + 16\,(8\,b_1 - 80\,b_2)^2 + 16\,(16\,a_2 - 160\,t)^2 - 4096\,(8\,b_1 - 80\,b_2)b_2 - 8192\,(16\,a_2 - 160\,t)t) - 720\,(4\,a_1 - 24\,a_2 + 120\,t)^7 (16\,a_2 - 160\,t) - 3600\,(4\,a_1 - 24\,a_2 + 120\,t)^3 (48\,a_2 - 1376\,t) - 720\,(4\,a_1 - 24\,a_2 + 120\,t)^5 (272\,a_2 - 3168\,t) - 2764800\,(8\,b_1 - 80\,b_2)b_2$$

and

$$d(x,t) = 2024+2123366400\,t^2+874800\,(16\,a_2-160\,t)^2+3720\,(4\,a_1-24\,a_2+120\,t)^8+120\,(4\,a_1-24\,a_2+120\,t)^{10}+
518400\,(16\,a_2-160\,t)^4+(1+(2\,x-12\,b_1+60\,b_2)^2+(4\,a_1-24\,a_2+120\,t)^2)^6+530841600\,{b_2}^2+356400\,(8\,b_1-80\,b_2)^2+
518400\,(8\,b_1-80\,b_2)^4+120\,(8\,b_1-80\,b_2)(2\,x-12\,b_1+60\,b_2)^9+46080\,b_2(2\,x-12\,b_1+60\,b_2)^7-82944000\,(16\,a_2-
160\,t)t+(-1440\,(4\,a_1-24\,a_2+120\,t)^4+720\,(4\,a_1-24\,a_2+120\,t)^5(16\,a_2-160\,t)+240\,(4\,a_1-24\,a_2+120\,t)^2(56+
135\,(8\,b_1-80\,b_2)^2-45\,(16\,a_2-160\,t)^2)+32400\,(4\,a_1-24\,a_2+120\,t)(16\,a_2-288\,t)+7200\,(4\,a_1-24\,a_2+120\,t)^3(48\,a_2-
544\,t)+3360+32400\,(8\,b_1-80\,b_2)^2-54000\,(16\,a_2-160\,t)^2+2764800\,(8\,b_1-80\,b_2)b_2+5529600\,(16\,a_2-160\,t)t)(2\,x-
12\,b_1+60\,b_2)^4+(-960\,(4\,a_1-24\,a_2+120\,t)^6(8\,b_1-80\,b_2)+57600\,(4\,a_1-24\,a_2+120\,t)^3(8\,b_1-80\,b_2)(16\,a_2-160\,t)-
43200\,(4\,a_1-24\,a_2+120\,t)^2(24\,b_1-272\,b_2)-7200\,(4\,a_1-24\,a_2+120\,t)^4(48\,b_1-448\,b_2)+345600\,b_1-5529600\,b_2-
86400\,(8\,b_1-80\,b_2)^3-86400\,(8\,b_1-80\,b_2)(16\,a_2-160\,t)^2+172800\,(4\,a_1-24\,a_2+120\,t)((8\,b_1-80\,b_2)(16\,a_2-160\,t)-
32\,(16\,a_2-160\,t)b_2+64\,(8\,b_1-80\,b_2)t))(2\,x-12\,b_1+60\,b_2)^3+(13440\,(4\,a_1-24\,a_2+120\,t)^6+240\,(4\,a_1-24\,a_2+
120\,t)^8-240\,(4\,a_1-24\,a_2+120\,t)^4(-326+45\,(8\,b_1-80\,b_2)^2-135\,(16\,a_2-160\,t)^2)+480\,(4\,a_1-24\,a_2+120\,t)^2(-76+
135\,(8\,b_1-80\,b_2)^2+1215\,(16\,a_2-160\,t)^2)-129600\,(4\,a_1-24\,a_2+120\,t)^3(32\,a_2-256\,t)-12960\,(4\,a_1-24\,a_2+
120\,t)^5(32\,a_2-256\,t)-64800\,(4\,a_1-24\,a_2+120\,t)(-96\,a_2+1280\,t+4\,(8\,b_1-80\,b_2)^2(16\,a_2-160\,t)+4\,(16\,a_2-
160\,t)^3)+12144-97200\,(8\,b_1-80\,b_2)^2+32400\,(16\,a_2-160\,t)^2+530841600\,{b_2}^2-33177600\,(16\,a_2-160\,t)t+
2123366400\,t^2)(2\,x-12\,b_1+60\,b_2)^2+(-360\,(4\,a_1-24\,a_2+120\,t)^8(8\,b_1-80\,b_2)-17280\,(4\,a_1-24\,a_2+120\,t)^5(8\,b_1-
80\,b_2)(16\,a_2-160\,t)-1440\,(4\,a_1-24\,a_2+120\,t)^6(8\,b_1-240\,b_2)+32400\,(4\,a_1-24\,a_2+120\,t)^4(8\,b_1+112\,b_2)+
64800\,(4\,a_1-24\,a_2+120\,t)^2(-40\,b_1+752\,b_2+4\,(8\,b_1-80\,b_2)^3+4\,(8\,b_1-80\,b_2)(16\,a_2-160\,t)^2)-777600\,(4\,a_1-24\,a_2+
120\,t)((8\,b_1-80\,b_2)(16\,a_2-160\,t)+64\,(16\,a_2-160\,t)b_2-128\,(8\,b_1-80\,b_2)t)-172800\,(4\,a_1-24\,a_2+120\,t)^3(3\,(8\,b_1-
80\,b_2)(16\,a_2-160\,t)+32\,(16\,a_2-160\,t)b_2-64\,(8\,b_1-80\,b_2)t)-648000\,b_1+8553600\,b_2+259200\,(8\,b_1-80\,b_2)^3+
1296000\,(8\,b_1-80\,b_2)(16\,a_2-160\,t)^2-33177600\,(8\,b_1-80\,b_2)^2b_2+33177600\,(16\,a_2-160\,t)^2b_2-132710400\,(8\,b_1-
80\,b_2)(16\,a_2-160\,t)t)(2\,x-12\,b_1+60\,b_2)+80\,(4\,a_1-24\,a_2+120\,t)^6(191+63\,(8\,b_1-80\,b_2)^2+27\,(16\,a_2-160\,t)^2)+
21600\,(4\,a_1-24\,a_2+120\,t)^3(-368\,a_2+3488\,t+4\,(8\,b_1-80\,b_2)^2(16\,a_2-160\,t)+4\,(16\,a_2-160\,t)^3)+240\,(4\,a_1-24\,a_2+
120\,t)^4(599+135\,(8\,b_1-80\,b_2)^2-225\,(16\,a_2-160\,t)^2-11520\,(8\,b_1-80\,b_2)b_2-23040\,(16\,a_2-160\,t)t)-16200\,(4\,a_1-
24\,a_2+120\,t)(496\,a_2-6240\,t+80\,(8\,b_1-80\,b_2)^2(16\,a_2-160\,t)+16\,(16\,a_2-160\,t)^3+4096\,(8\,b_1-80\,b_2)(16\,a_2-
160\,t)b_2-4096\,(8\,b_1-80\,b_2)^2t+4096\,(16\,a_2-160\,t)^2t)+24\,(4\,a_1-24\,a_2+120\,t)^2(3881+12150\,(8\,b_1-80\,b_2)^2+
28350\,(16\,a_2-160\,t)^2+691200\,(8\,b_1-80\,b_2)b_2+22118400\,{b_2}^2+88473600\,t^2)-120\,(4\,a_1-24\,a_2+120\,t)^9(16\,a_2-
160\,t)-2160\,(4\,a_1-24\,a_2+120\,t)^5(240\,a_2-4576\,t)-1440\,(4\,a_1-24\,a_2+120\,t)^7(80\,a_2-864\,t)+1036800\,(8\,b_1-
80\,b_2)^2(16\,a_2-160\,t)^2+(-120\,(4\,a_1-24\,a_2+120\,t)^2+360\,(4\,a_1-24\,a_2+120\,t)(16\,a_2-160\,t)+120)(2\,x-12\,b_1+
60\,b_2)^8+(480\,(4\,a_1-24\,a_2+120\,t)^2-240\,(4\,a_1-24\,a_2+120\,t)^4+960\,(4\,a_1-24\,a_2+120\,t)^3(16\,a_2-160\,t)+
2320+2160\,(8\,b_1-80\,b_2)^2+5040\,(16\,a_2-160\,t)^2-1440\,(4\,a_1-24\,a_2+120\,t)(64\,a_2-960\,t))(2\,x-12\,b_1+60\,b_2)^6+
(-720\,(4\,a_1-24\,a_2+120\,t)^4(8\,b_1-80\,b_2)-17280\,(4\,a_1-24\,a_2+120\,t)(8\,b_1-80\,b_2)(16\,a_2-160\,t)+4320\,(4\,a_1-
24\,a_2+120\,t)^2(8\,b_1-176\,b_2)-51840\,b_1+103680\,b_2)(2\,x-12\,b_1+60\,b_2)^5-24883200\,(8\,b_1-80\,b_2)b_2$$

is a solution to the $(NLS5)$ equation (1).

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