



Evaluating Properties and Performance of Long Memory Models from an Emerging Foreign Markets Return Innovations

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

The study investigate, evaluate properties and performance of long memory models from emerging foreign markets return innovations between 1991 – 2020. The purpose of the study include to; investigate the persistence of shocks in Nigerian international markets, model long-range dependence, test the efficient markets hypothesis using fractionally integrated volatility models, develop an appropriate long memory model for Nigerian international markets, compare the advantages between short and long memory models in modeling for the returns in Nigerian international Markets and Give forecast values for future occurrences. The design for the study was an ex post facto research design. The data used for this study were Nigerian crude oil prices (Dollar per Barrel), exchange rate, and Agricultural Commodity prices extracted from the website of the Central Bank of Nigeria (CBN) www.cbn.ng. The total data points were 1044 and it spanned from 1st January 1991 to 30th January 2020. The statistical software used for data analysis was STATA 15 and OX metrics version 7. In an attempt to achieve the aim of the study, parametric and non-parametric methods of detecting Long Memory were applied. The study applied short and long memory models in an attempt to spot out the deficiencies associated with the short memory models. The results confirmed the presence of long memory in sales and returns on prices in Nigerian international markets. The presence of long memory in both sales and returns on prices in Nigerian International markets disprove the efficient market hypothesis which says that the future returns and volatility values are unpredictable. Similarly, base on performance evaluation using the Akaike information criteria, ARFIMA(1,-0.021,1) model was found to be the best fit model to the data after checking the adequacy of the model selected. Sequel to the above, it was recommended that there is a need for a strong financial and economic reform policy to curb persistent shocks in Nigerian international

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markets. This is because a stable local financial currency builds confidence in an economy, especially when foreign investors intend to invest in the country's economy. For example, exchange rate policies also trim down the desire for local investors to trade in the international market. Also, for empirical estimation of long memory sales and returns on prices in Nigeria international markets, ARFIMA(1,-0.021,1) model should be considered appropriate. Two years (January, 20 to Dec-22) step ahead forecast shows that the predicted value for Cocoa Bean Sale using the ARFIMA (1,-0.021,1) falls between the range of 1.907247 to 1.915947.

Keywords: Evaluating; properties; performance; emerging; markets and innovation.

1 Introduction

1.1 Background to the study

Over the past several years, many researchers have been dedicated to developing and improving time series forecasting models, thereby drawing significant attention towards the applications of short memory models in several studies. Worthy of mention is a short memory model like the Autoregressive Integrated Moving Average (ARIMA) which is used when the time-series data to be estimated are stationary and without missing data. However, like every other technique for estimating financial data series, they have their drawbacks. The short memory model (for example, the Autoregressive Integrated Moving Average (ARIMA) has inadequacy due to its inability to forecast extreme cases or nonlinear relationships. Also, another drawback to the use of short memory models is the inability to model long-range dependence, excessive speculative interest, price instability in foreign markets, and lack of convergence between the future and present cash market. Many real-life problems have time-series data available on several related variables of interest. Therefore, analyzing and modeling them jointly helps to understand the underlying dynamic relationships among them. More importantly, is the fact that prices of commodities from emerging foreign markets return innovations have recently been confronted with the problems of volatility, long-range dependence, excessive speculative interest, price instability, a higher level of risks, shocks, steady increase in exchange rate hence making the US dollar other foreign currencies to be under pressure, resulting to its free-fall. This study, therefore, seeks to build models such as Autoregressive Fractional Integrated Moving Average (ARFIMA) and Fractional Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) models that will capture and estimates persistent shocks in the form of volatility clustering, develop an appropriate long memory model for modeling return innovations from emerging foreign markets, to select appropriate long memory model for forecasting long memory process of return innovations from emerging foreign markets and give forecast value its future price values..

2 Method of Analysis

2.1 Source of data used for the study

The present study used Nigerian exchange rate and agricultural commodity price data collected from the website of the central bank of Nigeria (CBN) www.cbn.ng. The data for this study was extracted from 1st January 1991 to 30th January 2020. The statistical software used were STATA 15 and OX metrics version 7

2.2 Detection of long memory

It is a well known fact that many financial time series are highly persistent, implying that an unforeseeable shock to the variable has long-lasting impacts. In this case, the absolute and squared returns of autocorrelation functions of the time series exhibit very slow decay. To detect the present highly persistent and long impact, the following test such as Rescaled Range (R/S) statistic, GPH, GPS test, and single break/change test will be conducted on Nigerian crude oil prices (Dollar Per Barrel), exchange rate, and agricultural commodity prices obtained from present Nigerian international markets.

2.2.1 Lo's R/S Statistics

The Rescaled Range (R/S) statistic was originally proposed [1] and later modified [2] in the year 1991. It remarked that the original statistics is not robust to short range dependence. Thus, modified the R/S statistics as follows:

$$Q_T = \frac{1}{\hat{\sigma}_T(q)} \left[\max_{1 \leq k \leq T} \sum_{t=1}^k (y_t - \bar{y}) - \min_{1 \leq k \leq T} \sum_{t=1}^k (y_t - \bar{y}) \right] \quad (2.1)$$

Where $\bar{y} = \frac{\sum_{i=1}^T y_i}{T}$ and $\hat{\sigma}_T(q)$ represent the square root of the Newey-West estimate of the long-run variance with bandwidth q .

2.2.2 GPH Test

The GPH was proposed as a semi-parametric approach to test for long memory [3] using the following regression,

$$\ln I(w_j) = \beta - d \ln \left[4 \sin^2 \left(\frac{w_j}{2} \right) \right] + n_j \quad (2.2)$$

Where $w_j = \frac{2n_j}{T}$, $j = 1, 2, \dots, n$; n_j the residual is term and w_j denotes Fourier frequencies. $I(w_j)$ represent the periodogram of a time series r_t and defined as:

$$I(w_j) = \frac{1}{2\pi T} \left| \sum_{t=1}^T r_t e^{-w_j t} \right|^2 \quad (2.3)$$

2.2.3 GPS test

The Gaussian semi-parametric estimate is done based on the whittle approximation maximum likelihood estimator [4]. GPS estimator can be written as;

$$\hat{d}_{GPS} = \arg \min_d R(d) \quad (2.4)$$

Where; $R(d) = \log \left(\frac{1}{m} \sum_{j=1}^m \lambda_j^{2d} I(\lambda_j) \right) - \frac{2d}{m} \sum_{j=1}^m \log \lambda_j$, also, m represents the bandwidth, [5] which increase with the sample size T . $I(\lambda_j)$ denotes the periodogram and $\lambda_j = 2\pi j / T$.

2.3 Model specification

In line with the specific objectives for this study, two classes of models were used in the study and they include: ARFIMA and FIGARCH models.

2.3.1 ARFIMA model specification

The ARFIMA model, according to Sanusi, et al. [6], Brown et al. [7], simply represent an acronym that stand for autoregressive fractional integrated moving average. The general form of an ARFIMA (p,d,q)

model as was defined Korkmaz et al. [8] can be stated as thus : Recall the left hand side of the ARIMA model.

$$\phi(B)(1-L)^d Y_t = \Theta(L^q)\varepsilon_t, \varepsilon_t \approx (0, \sigma_t^2) \quad (2.5)$$

$\phi(B)$ Is the Autoregressive Operator, b represents is the Moving Average Operator, d represents a fractional integration real number parameter, L denotes the lag operator and ε_t is a white noise residual; $(1-L)^d$ stands for the fractional differencing lag operator. Considering two cases of the “ d ”, when $d = 0$ and $d = 1$, Granger et al. [9] the “ d ” used in the model lies as thus: $0 \leq d \leq 1$. Sanusi et al[6]. noted that the ARFIMA model has slow (hyperbolic) decay patterns in the Auto-Correlation Function (ACF). These characteristics of having slow decay patterns revealed long memory in terms of shocks. According to Hosking [10] observation d lies between $(-0.5$ and $0.5)$ i.e. $-0.5 < d < 0.5$, and it was used by Kang & Yoon [11] & Korkmaz et al [8] and when $-0.5 < d < 0.5$, the series is said to be stationary. In this case, the effect of market shocks to (ε_t) decay at a gradual rate to zero. Also, when $d=0$, the series is said to have short memory and the effect of shocks to ε_t decays at geometrical rate whereas when $d=1$, there is the presence of a unit root process. John and Jo-Hui, [12] revealed that long memory or positive dependence exist among distant observations when $0 < d < 0.5$. They further opined that the series has intermediate memory or anti-persistence when $-0.5 < d < 0$, non-stationary when $d \geq 0.5$ and stationary when $d \leq 0.5$. Also, there is need for us to consider a non-invertible process and this simply means that the series cannot be determined by any auto-regressive model. According to Margyan and Ramanathan [13], the autoregressive fractional integral moving Average (ARFIMA) model is nothing but the fractional integrated part of ARMA model

2.3.2 FIGARCH model

The FIGARCH (p,d,q) model on the other hand, is an acronym that stands for fractional integrated General autoregressive conditional Heteroskedasticity. According to Maryan and Ramanathan [13], the FIGARCH (p,d,q) model was introduced by Bordanon et al[14] with the intension to capture seasonal which allows for both periodic patterns and long memory characteristic in the conditional variance. The model merges both periodic and long memory component:

$$h_t = \alpha_0 + \alpha(L) \varepsilon_t^2 + \beta(L)h_t + \left[1 - (-1L^s)^d\right] \varepsilon_t^2 \quad (2.6)$$

According to Maryam and Ramanathan [13], the first three terms in the conditional variance produce the general model; the fourth term introduces a long memory component which takes the zero and seasonal frequencies. Also, the parameter “ s ” represents the length of the cycle, while d stands for the degree of memory. Rearranging the terms in model (3.3), an alternative specification model for the FIGARCH (p,d,q) model may be obtained as thus.

$$[L - \beta(L)]h_t = \alpha_0 + [1 - \beta(L)(1-L)^d] \varepsilon_t^2 \quad (2.7)$$

From equation (2.4), the conditional variance h_t of x_t is given as

$$h_t = \alpha_0 [1 - B(1)]^{-1} + [1 - B(L)]^{-1} \phi(L)(1-L)^d \varepsilon_t^2 \quad (2.8)$$

$$h_t = \delta_t^2 = \alpha_0 (1 - \beta(D))^+ + \lambda(L) \varepsilon_t^2$$

By replacing $p_l - (1 - B(L)J^1\phi(L) (1 - L)^d]$ as $\lambda(L)$, with $0 \leq d \leq 1$. Where $\lambda(L) = \lambda_1 + \lambda_2 L^2 + \dots$ of course, for the FIGARCH (p,d,q) for the above model to be the ARCH (∞) representation in (3.5) must be non-negative ie $\lambda_k \geq 0$ for $k = 1,2$ some sufficient conditions

2.4 Parameter estimation for ARFIMA model:

The Gewek and porter-Hudak [3] procedure for estimating parameters of ARFIMA model is defined as thus: Given a spectral density function of stationary series

$$x_t \tau t = 1, \dots, T \quad F_x(\lambda) \left[4sm^2 \left(\frac{\lambda}{2} \right) \right]^{-d} f_\varepsilon(\lambda) \dots \dots \quad (2.9)$$

Where $f_\varepsilon(\lambda)$ is the spectral density of ε_t , assumed to be a finite and continuous function on the interval $(-\pi, \pi)$. Taking logarithm of the spectral density function $F_y(\lambda)$, we have

$$\text{Log}(f_y(\lambda)) = \log(f_\varepsilon(0)) - d \log \left[4 \sin^2 \left(\frac{\lambda}{2} \right) \right] + \log \frac{f_\varepsilon(\lambda)}{f_\varepsilon(0)} \quad (2.10)$$

Supposing $I_y(\lambda_j)$ is the periodogram to be evaluated using fourier series, given as $\lambda_j = \frac{2\pi}{T}$ where $j = 1, 2, \dots, m$, and τ is the number of observations and m is the number of consider fourier frequencies. The number of periodogram ordinates which will be used in the regressor is given as thus:

$$\text{Log}[I_x(\lambda)] = \log[f_\varepsilon(0)] - d \log \left[4 \sin^2 \left(\frac{\lambda}{2} \right) \right] + \log \frac{f_\varepsilon(\lambda)}{f_\varepsilon(0)} + \log \frac{I_x(\lambda_j)}{f_x(\lambda_j)} \dots \dots \quad (2.11)$$

Where $\log[f_\varepsilon(0)]$ is a constant, $\log[4 \sin^2 \{N^2\}]$ is the exogenous variable and $\text{Log} \left[\frac{I_x(\lambda_j)}{f_x(\lambda_j)} \right]$ is a disturbance error. Geweke and Porter-Hudark [3], consider two major assumptions related to asymptotic behavior of the model in equation (2.10) and the assumptions are as thus: H_1 : for low frequencies, supposing the $\text{Log} \left[\frac{f_\varepsilon(\lambda)}{f_\varepsilon(0)} \right]$ is negligible versus the alternative hypothesis (H_2): The random variables $\text{Log} \left[\frac{I_x(\lambda_j)}{f_x(\lambda_j)} \right]$ such that $J=1, 2, \dots, m$ are asymptotically and identically distributed under the hypothesis H_1 , and H_2 [15]. We can write the linear regression as:

$$\text{Log}[I_x(\lambda)] = \alpha - d \left(\log \left(4 \sin^2 \left\{ \frac{\lambda_j}{2} \right\} \right) \right) + \text{lj} \quad (2.12)$$

Where; $\ell_j \approx$ identically independently distributed with $(-c, 2\pi/6)$. Given $y_j = -\log(4 \sin^2 \left\{ \lambda_j / 2 \right\})$ the Geweke and porter –Hudak [3] estimate of the regression $\log [I_x(\lambda)]$ on the constant α and y_j .

The estimate of d is given by the equation below:

$$dGPH = \frac{\sum_{j=1}^m (x_j - \bar{x}) \log [I_x(\lambda_j)]}{\sum_{j=1}^m (\lambda_j - \bar{X})^2} \tag{2.13}$$

Where; $\bar{X} = \frac{\sum_{j=1}^m X_j}{m}$. Furthermore, it revealed that under the assumption of normality for given data series it has been proved that the result of the estimate is consistent and asymptotically normal [15].

2.5 Parameter estimation for FIGARCH model

The likelihood of a FIGARCH (p,d,q) process based on the series $\{\omega_1, \omega_n, \dots, \varepsilon\tau\}$ may be written as:

$$\text{Log } h(\theta_1, \omega_1, \omega_2, \dots, \varepsilon\tau) \cong -0.5T \text{Log}(2\pi) - 0.5 \sum_{t=1}^T [\log(ht) + \xi_t^2 h^{-1}] \tag{2.14}$$

Where; $\theta^1 \equiv (\alpha_0, d, \beta_1, \dots, \beta_p, \phi_1, \dots, \phi_q)$.

The likelihood function is said to be maximized conditional on the startup values. Specifically there is need to fix all the pre-sample value of ε_t^2 for $t=0, -1, -2, \dots$. In the infinite ARCH representation in the model in equation (2.5) at the unconditional sample variance and according to Maryan & Rammanathan [13] for the FIGARCH (p,d,q) model with $d > 0$, the population variance does not exist. Therefore, in most practical applications with high frequency data, the standardized innovations $Z_t = h_t^{-1/2} \varepsilon_t$ are leptokurtic and normally distributed through time. Maryan & Ramandathan [13] suggested that in these situations the robust quasi- maximum likelihood estimator (QMLE) methods discussed in Wess [16] and Bollerslev & Wooldridge [17] may give better estimate in terms of inference.

3 Results



Fig. 1. Time plot for Nigeria Crude Oil Price (Dollar/Barrel)



Fig. 2. Time plot on monthly coco bean price (Dollar)



Fig. 3. Time plot on monthly exchange rate (Naira/Dollar)

Fig. 1 -3 which are plots of the raw data and time (years). On the horizontal axis we have the raw data while on the vertical axis is in time (years). This portrays the direction and moving trend of the variable under study.

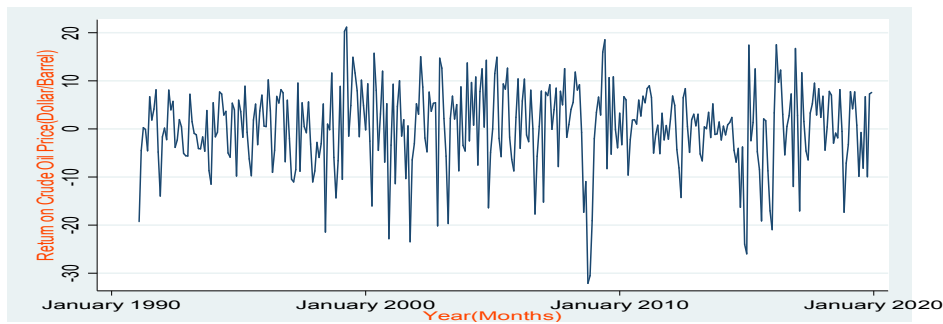


Fig. 4. Time plot of the monthly returns on Nigeria Crude Oil Price (Dollar/Barrel)

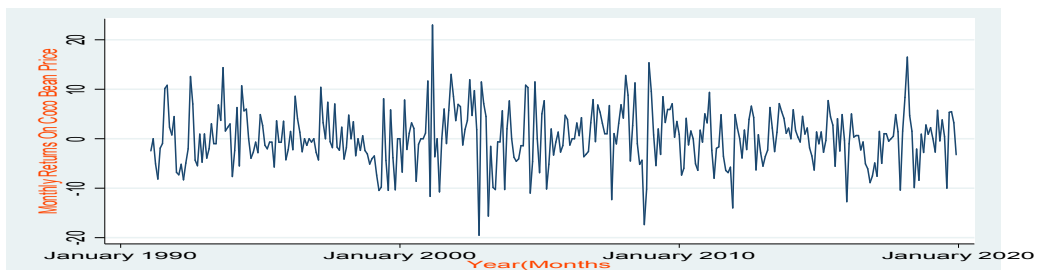


Fig. 5. Time plot of the monthly returns on Coco Bean Price (Dollar)

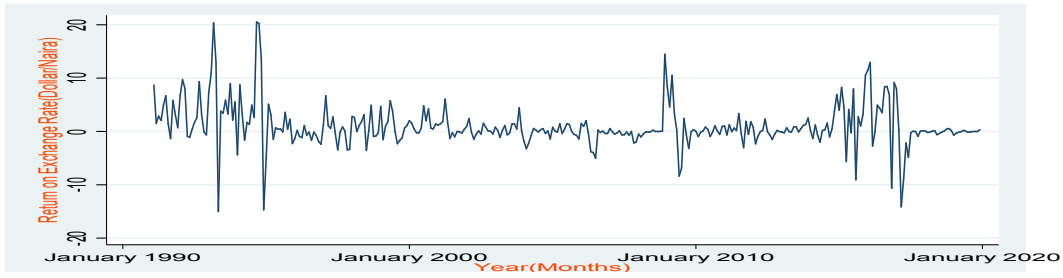


Fig. 6. Time plot on the monthly returns on exchange rate (Dollar/Naira)

This is followed by Fig. 4-6 which are plots of returns on the raw data and time (years). On the horizontal axis we have the returns raw data while on the vertical axis is in time (years).

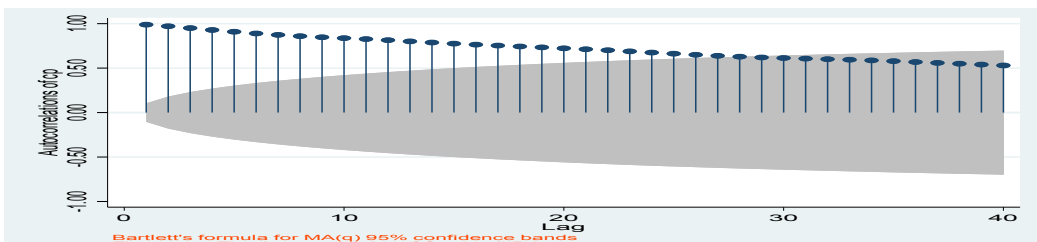


Fig. 7. The autocorrelation of Crude Oil Price

The Partial Function of Crude Price International Markets

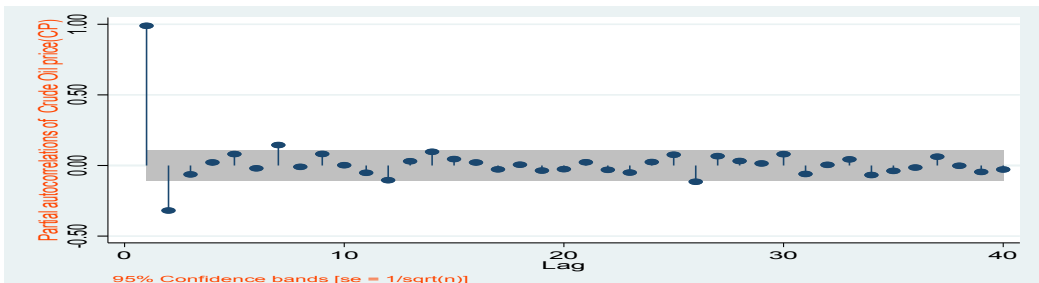


Fig. 8. Partial autocorrelations of Crude Oil Price (CP)

The Autocorrelation Function of Coco Bean Price International Markets

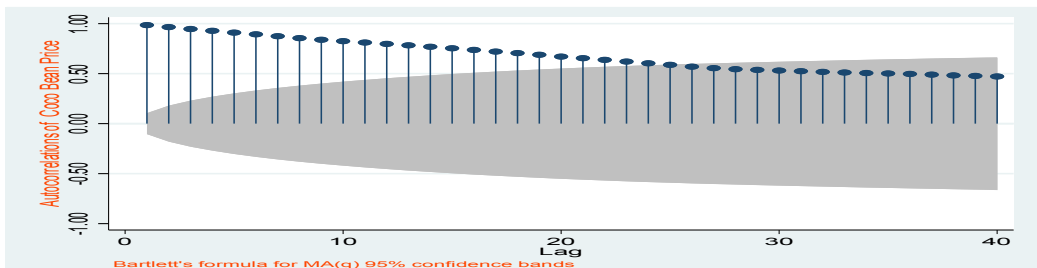


Fig. 9. Autocorrelation of Coco Bean Price

The Partial Function of Cocoa Bean Prices in International Markets

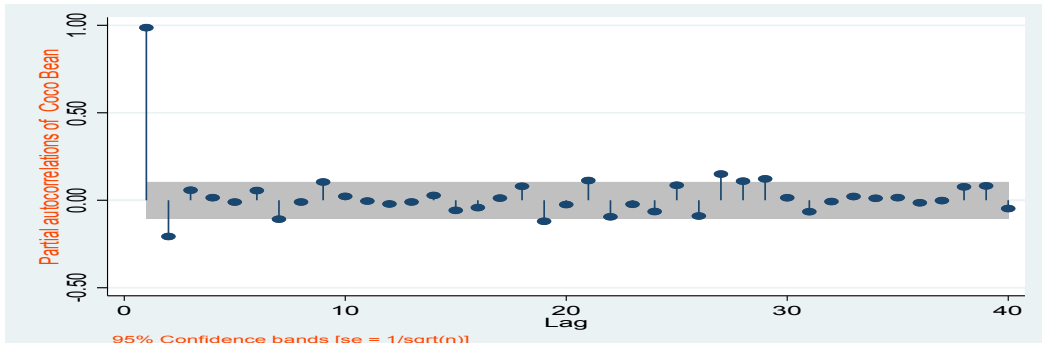


Fig. 10. Partial Autocorrelations of Coco Bean

The Autocorrelation Function of Exchange Rate in International Markets

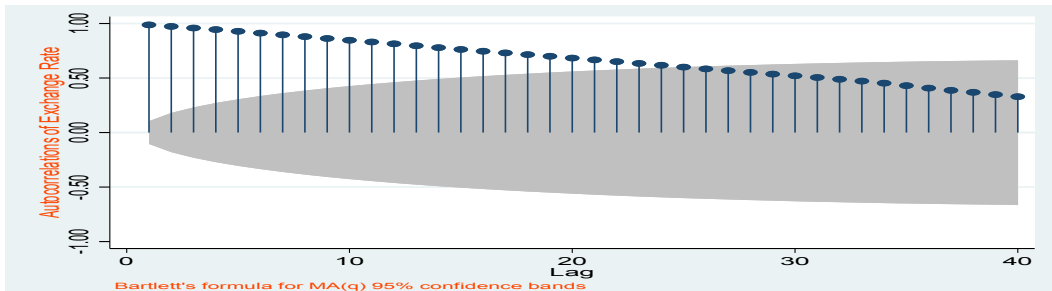


Fig. 11. Autocorrelations of exchange rate

The Partial Function of Exchange Rate Prices in International Markets

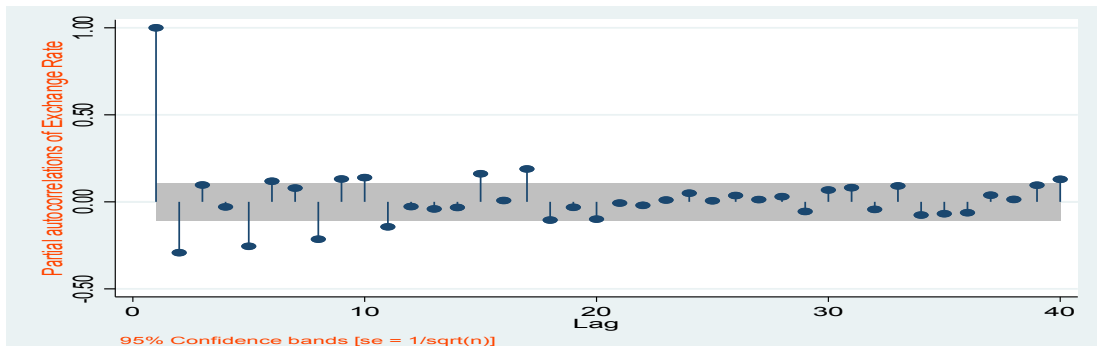


Fig. 12. Partial Autocorrelations of exchange rate

The Fig. 7, 9 and 11 above are the correlogram of Autocorrelation Function (ACF) of crude oil prices, cocoa beans prices and exchange rate (Naira Per Dollar) for lag 40 respectively, while their Partial Correlation Function (PACF) are represented in Fig. 8, 10 and 12.

Table 1. Descriptive statistics of the returns on prices in international markets

Variable	Mean	Median	Max	Min	Std.D	Skewness	Kurtosis	Jarq. Bera	P-value
RCP	0.295	0.766	21.158	32.105	8.592	-0.679	3.971	40.320	0.000
RCB	0.195	0.000	22.986	-19.513	5.753	0.053	3.891	11.654	0.003
REXCH	1.008	0.169	20.538	-15.007	4.0129	1.026	9.618	694.057	0.000

The results were all tested Significant at the 1%, 5% and 10% level of significance respectively

Table 2. Unit root test for stationarity at first difference

Variable(s)	Augmented Dickey Fuller Test (ADFT)			Phillip PerronTest (PPT)			KPSS Test			
	Test Stat			Test Stat			Test Stat			
	1%	5%	10%	1%	5%	10%	1%	5%	10%	
CP	-10.261	-2.337	-1.649	-12.84	-13.371	-3.986	0.591	0.216	0.146	0.119
CB	-12.502	-2.337	-1.649	-1.284	-15.144	-3.986	0.385	0.216	0.146	0.119
EXCH	-12.162	-2.337	-1.649	-1.284	-13.815	-3.986	0.724	0.216	0.146	0.119

The results were all tested Significant at the 1%, 5% and 10% level of significance respectively

Table 2 shows the results of the Unit Root Test and this was done using the Augmented Dickey-Fuller test, Phillip Perron Test statistics and KPSS Test. The Augmented Dickey-Fuller (ADF) test has the following hypothesis: H_0 : Unit root exists (non-stationary) versus H_1 : No unit root exists (stationary). Similarly, the stationarity of the variables were also tested using the Phillip Perron Test and the KPSS test statistics. For the Augmented Dickey-Fuller (ADF) test method, the decision is based on when the null hypothesis (H_0), at P- value 95% confidence level is greater than the standard probability value of 0.05. We reject the null hypothesis concluding that the variable under considerable within the period of the study is not stationary. Similarly, the Phillip Perron statistics were tested at P- value 95% confidence level as against the standard probability value of 0.05 and if the calculated probability is greater than the standard probability value of 0.05, we reject the null hypothesis concluding that the variable under considerable within the period of the study is not stationary. This follows the conclusion that the variable required further differencing to render the series stationary.

Table 3. Estimate of the L Jung-Box (Q) Statistic

Sales		Q-statistic	Probability	Returns on Prices	Q-statistic	Probability
Crude oil price(CP)	Q(5)	1592.717	0.000	Q ^S (5)	11.761	0.039
	Q(10)	2922.286	0.000	Q ^S (10)	21.254	0.019
	Q(20)	5109.223	0.000	Q ^S (20)	31.490	0.049
	Q(50)	8984.009	0.000	Q ^S (50)	64.395	0.083
Coco Bean(CB)	Q(5)	1584.439	0.000	Q ^S (5)	9.781	0.0817
	Q(10)	2901.609	0.000	Q ^S (10)	13.173	0.214
	Q(20)	4932.883	0.000	Q ^S (20)	28.224	0.049
	Q(50)	8082.934	0.000	Q ^S (50)	83.250	0.002
Exchange Rate(ER)	Q(5)	1622.64	0.000	Q ^S (5)	49.135	0.000
	Q(10)	3008.647	0.000	Q ^S (10)	66.349	0.000
	Q(20)	5106.029	0.000	Q ^S (20)	140.357	0.000
	Q(50)	7404.828	0.000	Q ^S (50)	160.560	0.000

The results were all tested at 1%, 5% and 10% level of Significance respectively

Table 3 contains estimate of the L Jung-Box (Q) Statistic used in testing the presence of Autoregressive Conditional Heteroskedasticity (ARCH) effect. The presence of Autoregressive Conditional Heteroskedasticity (ARCH) is a condition which shows that a series violate the assumption of homoskedasticity, i.e., not having constant variance. Table 4 contains the estimate for test of the presence of long memory in Nigerian international markets for both raw price and sales, and returns on price and sales

Table 4. Estimates for the test of the presence of long memory in Nigerian international markets

Raw Prices Test Stat	Crude Oil Price (CP)	Exchange Rate (Exch)	Cocoa Bean (CB)
Lo's R/S Test	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]
GPH Test			
M = T ^{0.5}	[0.803***]	[1.028***]	[1.123***]
M = T ^{0.6}	[0.781***]	[0.991***]	[0.987***]
M = T ^{0.7}	[0.981***]	[0.956***]	[1.002***]
M = T ^{0.8}	[1.097***]	[1.014***]	[1.000***]
Robinson Estimates			
0.5	[0.814***]	[1.038***]	[1.083*]
0.6	[0.777***]	[0.987***]	[0.982*]
0.7	[0.962***]	[0.939***]	[0.983*]
0.8	[1.059***]	[0.976***]	[0.960*]
Returns on Prices/Sales			
Lo's R/S Test	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]	[0.86, 1.747] [0.809, 1.862] [0.721, 2.098]	[0.861, 1.747] [0.809, 1.862] [0.721, 2.098]
GPH Test			
M = T ^{0.5}	[- 0.136]	[0.091]	[0.388]
M = T ^{0.6}	[- 0.124]	[- 0.080]	[0.238]
M = T ^{0.7}	[- 0.030]	[- 0.086]	[0.099]
M = T ^{0.8}	[- 0.095]	[- 0.030]	[0.165]
Robinson Estimates			
0.5	[-0.089]	[0.093]	[0.358]
0.6	[-0.123]	[-0.081]	[0.237]
0.7	[-0.027]	[-0.087]	[0.092]
0.8	[0.094]	[-0.029]	[0.159]

The results were all tested Significant at 1%, 5% and 10% respectively ie *, **and *** represents each level of significance

Table 5 contains the estimates for the applications of Short Memory (ARIMA) models in modeling price and sales and returns on prices in Nigerian international mark

Table 5. Estimate of the applications of short memory models in modeling price and sales, and returns on price and sales in Nigerian international markets

Sales	Models	Model Parameters				AIC	BIC	Least AIC/BIC
		α	Φ	β	Υ			
CP	ARIMA(1,1,1)	26.198 (0.015)	0.992 (0.000)	0.273 (0.000)	21.118 (0.000)	2057.02	2072.428	
	ARIMA(2,1,2)	25.046 (0.022)	0.978 (0.000)	0.276 (0.00)	55.982 (0.000)	2396.293	2411.70	
CB	ARIMA(1,1,1)	10.259 (0.908)	1.004 (0.000)	0.298 (0.000)	60.030 (0.000)	2420.589	2435.998	
	ARIMA(2,1,2)	10.284 (0.914)	(1.008) (0.000)	0.077 (0.000)	169.570 (0.000)	2781.955	2797.364	
ExchRate	ARIMA(1,1,1)	1.261 (0.022)	0.993 (0.000)	0.217 (0.003)	0.012 (0.000)	-535.485	-520.076	-535.485
	ARIMA(2,1,2)	1.261 (0.016)	0.987 (0.000)	0.067 (0.100)	0.031 (0.000)	-217.391	-201.982	-217.391
Returns on Prices RCP	ARIMA(1,1,1)	0.287 (0.627)	0.186 (0.522)	-0.021 (0.945)	71.571 (0.000)	2474.692	2490.101	
	ARIMA(2,1,2)	0.293 (0.564)	-0.942 (0.000)	0.972 (0.000)	72.999 (0.000)	2481.528	2496.937	
RCB	ARIMA(1,1,1)	0.192 (0.580)	-0.062 (0.850)	0.200 (0.535)	32.394 (0.000)	2199.599	2215.008	
	ARIMA(2,1,2)	0.195 (0.525)	0.719 (0.993)	-0.077 (0.993)	33.006 (0.000)	2206.086	2221.495	
RExchRate	ARIMA(1,1,1)	1.024 (0.006)	0.205 (0.041)	0.161 (0.124)	14.0868 (0.000)	1910.035	1926.043	1910.035
	ARIMA(2,1,2)	1.092 (0.035)	0.901 (0.000)	0.833 (0.000)	15.688 (0.000)	1947.975	1963.384	

The results were all tested Significant at the 1%, 5% and 10% level of significance respectively

Table 6 contains the estimates of the applications of long memory (ARFIMA) models in modeling price and sales, and returns on prices in Nigerian international markets.

Table 7 contains the estimates of the applications of long memory (FIGARCH) models in modeling price and sales, and returns on price and sales in Nigerian international markets

Table 6. Estimate of the applications of long memory models in modeling price and sales and returns on prices and sales in Nigerian international markets

Raw Sales	Models	α	Φ	β	d	ξ	AIC	BIC	Least AIC
CP	ARFIMA	49.722	0.849	0.011	0.431	20.695	2076.37	2076.37	
	(1, 0.43,1)	(9.128)	(0.000)	(0.889)	(0.000)	(0.000)			
	ARFIMA	48.912	0.616	0.061	0.487	3.556	2227.974	2247.235	
	(2, 0.487,2)	(0.686)	(0.000)	(0.454)	(0.000)	(0.000)			
CB	ARFIMA	1.917	0.984	0.236	-0.021	0.012	-531.204	-511.943	-531.204
	(1,-0.021,1)	(0.000)	(0.000)	(0.003)	(0.790)	(0.000)			
	ARFIMA	1.917	0.783	-0.337	0.497	0.018	-384.565	-365.304	-365.304
	(2,0.497,2)	(0.522)	(0.000)	(0.000)	(0.000)	(0.000)			
ExchRate	ARFIMA	171.640	0.995	0.249	0.075	60.156	2429.254	2448.512	
	(1, 0.075,1)	(0.120)	(0.000)	(0.000)	(0.214)	(0.000)			
	ARFIMA	166.143	0.919	-0.359	0.497	100.225	2611.047	2630.308	
	(2, 0.497,2)	(0.775)	(0.000)	(0.000)	(0.000)	(0.000)			
Returns on Prices									
RCP	ARFIMA	0.368	0.584	-0.196	-0.233	71.025	2474.293	2493.554	
	(1,-0.233,1)	(0.168)	(0.004)	(0.127)	(0.174)	(0.000)			
	ARFIMA	0.239	0.649	-0.752	0.154	71.410	2476.03	2496.38	
	(2, 0.154,2)	(0.759)	(0.000)	(0.000)	(0.003)	(0.000)			
RCB	ARFIMA	0.210	0.400	-0.134	-0.135	32.328	2201.009	2220.27	
	(1, -0.135,1)	(0.335)	(0.425)	(0.695)	(0.525)	(0.000)			
	ARFIMA	0.178	0.156	-0.217	0.100	32.328	2204.267	2223.528	
	(2, 0.100,2)	(0.720)	(0.805)	(0.728)	(0.052)	(0.000)			
RExchRate	ARFIMA	1.067	0.049	0.220	0.093	13.978	1910.149	1929.41	1910.149
	(1, 0.093,1)	(0.014)	(0.777)	(0.106)	(0.146)	(0.000)			
	ARFIMA	1.169	-0.898	0.810	0.276	13.923	1909.945	1928.706	1909.945
	(2, 0.276,2)	(3.145)	(0.000)	(0.000)	(0.000)	(0.000)			

The results were all tested Significant at 1%, 5% and 10% respectively

Table 7. Estimates of the applications of long memory (FIGARCH) models in modeling price and sales, and returns on price and sales in Nigerian international market

Sales						
Model Parameters	Crude Oil Price		Exchange Rate		Cocoa Beans	
	FIGARCH (1,d,1)	FIGARCH (2,d,2)	FIGARCH (1,d,1)	FIGARCH (2,d,2)	FIGARCH (1,d,1)	FIGARCH (2,d,2)
CST(M)	19.222 (0.000)	0.036 (0.000)	100.00 (0.000)	88.008 (0.000)	1.563 (0.000)	0.810 (0.000)
CST (V)	1.136 (0.000)	0.000 (0.000)	0.611 (0.8260)	99.990 (0.000)	0.010 (0.000)	0.000 (0.000)
d-FIGARCH	0.001 (0.000)	0.000 (0.000)	1.000 (0.000)	0.961 (0.000)	0.716 (0.000)	0.190 (0.000)
ARCH(α_1)	1.000 (0.000)	0.964 (0.000)	0.727 (0.000)	0.123 (0.000)	0.722 (0.000)	0.000 (0.000)
ARCH(α_2)		0.000 (0.000)		0.010 (0.000)		0.000 (0.000)
GARCH (β_1)	-0.000 (0.000)	0.000 (0.000)	0.794 (0.000)	0.099 (0.000)	0.258 (0.000)	0.000 (0.000)
GARCH (β_2)		0.000 (0.000)		0.001 (0.000)		0.000 (0.000)
Loglikelihood	-1430.18	-1784.29	-1823.05	-8678.583	-200.82	-501.446
Means (μ)	51.097	51.020	1.009	153.366	1.948	1.946
Skewness	-1.27	0.719	-0.907	-1.109	0.432	-0.627
Kurtosis	0.705	2.321	-1.027	-0.092	2.029	2.031
Jarque-Bera	100.47	104.88	62.818	71.451	22.918	56.683
AIC	8.272	8.362	10.536	10.472	1.186	2.928
SIC	8.327	8.372	10.592	10.550	1.242	3.006

Sales						
Model Parameters	Crude Oil Price		Exchange Rate		Cocoa Beans	
Returns on Prices						
CST(M)	0.204 (0.791)	0.010 (0.000)	0.173 (0.415)	0.010 (0.000)	0.123 (0.673)	0.010 (0.000)
CST (V)	69.087 (0.778)	36.653 (0.000)	70.940 (0.003)	7.996 (0.000)	37.599 (0.009)	16.437 (0.000)
d-FIGARCH	0.354 (0.966)	0.450 (0.000)	0.180 (0.002)	0.450 (0.000)	0.282 (0.013)	0.450 (0.000)
ARCH(α_1)	0.000 (1.000)	0.100 (0.000)	0.960 (0.000)	0.100 (0.000)	0.000 (1.000)	0.100 (0.000)
ARCH(α_2)		0.001 (0.000)		0.001 (0.000)		0.001 (0.000)
GARCH (β_1)	0.188 (0.989)	0.400 (0.000)	0.634 (0.000)	0.400 (0.000)	0.095 (0.969)	0.400 (0.000)
GARCH (β_2)		0.001 (0.000)		0.001 (0.000)		0.001 (0.000)
Loglikelihood	-0.679	-156	-855.482	-215	-1087.88	-685
Means (μ)	0.296	0.296	1.009	1.008	0.19507	-1.295
Skewness	-0.689	-4.448	2.7108	-5.652	0.017	42.197
Kurtosis	0.366	55.753	17.973	106.41	0.440	42.197
Jarque	29.377	46087.	5095.2	1.6555e+05	2.813	25842.
AIC	8.276	897	4.960 ^{****}	124	6.299	395
SIC	8.331	897	5.015 ^{****}	124	6.355	395

The results were all tested Significant at 1%, 5% and 10% respectively

Table 8 shows the estimates of the model selection criteria using the AIC and SIC of the short and long memory models

Table 8. Estimates of the model selection criteria using the AIC and SIC of the short and long memory models

Short Memory	Model	AIC	SIC	Least AIC	
Sales	CP	ARIMA(1,1,1)	2057.02	2072.428	
		ARIMA(2,1,2)	2396.293	2411.70	
	CB	ARIMA(1,1,1)	2420.589	2435.998	
		ARIMA(2,1,2)	2781.955	2797.364	
	ExchRate	ARIMA(1,1,1)	-535.485	-520.076	-535.485
		ARIMA(2,1,2)	-217.391	-201.982	
Returns on Prices	RCP	ARIMA(1,1,1)	2474.692	2490.101	
		ARIMA(2,1,2)	2481.528	2496.937	
	RCB	ARIMA(1,1,1)	2199.599	2215.008	
		ARIMA(2,1,2)	2206.086	2221.495	
	RExchRate	ARIMA(1,1,1)	1910.635	1910.635	1910.635
		ARIMA(2,1,2)	1947.975	1963.384	
Long Memory Models					
Sales	CP	ARFIMA(1, 0.43,1)	2076.37	2076.37	
		ARFIMA(2, 0.487,2)	2227.974	2247.235	
	CB	ARFIMA(1,-0.021,1)	-531.204	-511.943	-531.204
		ARFIMA(2,0.497,2)	-384.565	-365.304	
	ExchRate	ARFIMA(1, 0.075,1)	2429.254	2448.512	
		ARFIMA(2, 0.497,2)	2611.047	2630.308	
Returns on Prices	CP	FIGARCH(1,0.001,1)	8.272	8.327	
		FIGARCH(2,0.000,2)	8.362	8.372	
	CB	FIGARCH(1,0.716,1)	1.186	1.242	
		FIGARCH(2,0.190,2)	2.928	3.006	
	ExchRate	FIGARCH(1,1.000,1)	10.536	10.592	
		FIGARCH2,0.961,2)	10.472	10.550	
	RCP	ARFIMA(1,-0.233,1)	2474.293	2493.554	
		ARFIMA(2, 0.154,2)	2476.03	2496.38	
	RCB	ARFIMA(1, 0.135,1)	2201.009	2220.27	
		ARFIMA(2, 0.100,2)	2204.267	2223.528	
	RExchRate	ARFIMA(1, 0.093,1)	1910.149	1929.41	
		ARFIMA(2, 0.276,2)	1909.945	1928.706	
RCP	FIGARCH(1,0.354,1)	8.276	8.331		
	FIGARCH(2,0.45,2)	897	897		

Short Memory	Model	AIC	SIC	Least AIC
RCB	FIGARCH(1,0.282,1)	6.299	6.355	
	FIGARCH(2,0.450,2)	395	395	
RExchRate	FIGARCH(1,0.180,1)	4.960	5.015	4.960
	FIGARCH(2,0.450,2)	124	124	

The results were all tested Significant at 1%, 5% and 10% respectively

The overall best fitted short memory model: The overall best fitted Short Memory model for both sales and returns on Prices in Nigerian International markets was ARIMA (1,1,1) on sales (Exchange Rate)

$$\text{ARIMA (1,d,1)} \quad y = \varepsilon_t - 0.217 \varepsilon_{t-1} + 1.2610 y_{t-1} \quad 3.1$$

$$(0.000)(0.022)$$

$$\text{AIC}=-535.485, \text{BIC}= -520.076$$

The overall best fitted short memory model for both sales and returns on prices in Nigerian International markets was ARIMA (1,1,1) on Returns (Rexch rate)

$$\text{ARIMA (1,d,1)} \quad y = \varepsilon_t - 0.161 \varepsilon_{t-1} + 1.024 y_{t-1} \quad 3.2$$

$$(0.000) \quad (0.022)$$

$$\text{AIC}=1910, \text{BIC}= 1963.384$$

The Overall Best Fitted Long Memory Model: The overall best fitted long memory model for both sales and returns on prices in Nigerian international markets using ARFIMA models was an ARFIMA(1,-0.021,1) on sales (Coco Beans):

$$\text{ARFIMA (1,-0.021,1)} = \frac{0.984(D)(1-D)^{-0.021}(1-\mu)}{(0.000)} = \frac{0.236(D)\varepsilon_t}{(0.000)} \quad 3.3$$

$$\text{AIC} = -531.204 \quad \text{BIC} = -511.943$$

The overall best fitted long Memory model for both on Returns on Prices in Nigerian International markets using FIGARCH model was an FIGARCH (1,0.180,1) on returns(RExch Rate):

$$\text{FIGARCH(1,0.180,1)} (\sigma_t^2) = 0.173* + 0.960(1-0.415L)^{-1}(1-L)^{-0.180}(1-0.00L)(0.634)^2(z_{t-1}^2 - 1) \quad 3.4$$

$$\text{AIC} = 4.960, \text{BIC}=5.015$$

3.1 Test for ARCH effect

Table 9 shows estimation for ARCH effect using the Ljung–Box test for model adequacy following the hypothesis below: $H_0: \alpha_i = \text{Absence of ARCH effect up to order } q \text{ versus } H_1: \alpha \neq 0 \text{ for some } i \text{ elements ranging from } \{1, \dots, q\}$ such that the least one variable has presence of ARCH effect. The decision to accept or reject the hypothesis was based on the probability value ,ie, if the probability value is greater than the 5% level of significance then the null hypothesis will be accepted. However, if we fail to accept the null hypothesis then the alternative hypothesis will be accepted. Also, the row in Table 10 contains the result for normality test using Jarque-Bera test. The hypothesis states that $H_0: \alpha_i$ is normally distributed versus $H_1: \alpha$ is not normally distributed. The decision to accept or reject the hypothesis was based on the probability value,i.e, if the probability value is greater than the 5% level of significance then the null hypothesis will be accepted . However, if we fail to accept the null hypothesis then the alternative hypothesis will be accepted .

Table 9. Summary of diagnostic check for ARFIMA (1,-0.021,1) model

Test	(X ²)	P-value
LJung-Box	9.7814	0.0817
Jarque-Bera	7.09	0.0288

The results were all tested Significant at the 1%, 5% and 10% level of significance respectively

Table 9 contains the summary of diagnostic check for ARFIMA(1,-0.021,1) model. This was done using the LJung Box and Jargue Bera test statistics

3.2 The Quantile-Quantile Test (Q-Q plot)

The Q-Q plot is a scatter plot plotted between two sets of quantiles against each another. This is done to examine whether the two sets of quantiles came from the same distribution and if that is true then the two lines lie straight on each other, revealing that the residual of the estimated model follow a standardized order of a normal distribution

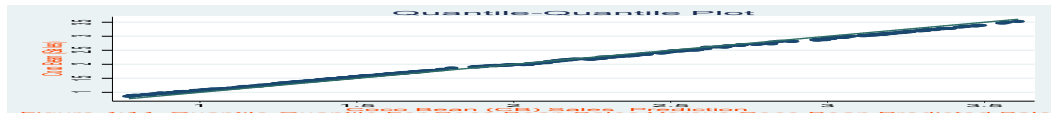


Fig. 13. Quantile-quantile for coco bean sales versus coco bean predicted sales

3.3 Econometric criteria and forecasting

This involves evaluating the Performance of the model that was chosen, ARFIMA (1,-0.021,1), after checking the adequacy of the model selected. The result of the forecast is shown in the Table 10.

Table 10. Forecasting performance of ARFIMA (1,-0.021,1)

Date	Predicted	Observed
Jan-21	1.910879	0.69891
Feb-21	1.911147	0.698912
Mar-21	1.911411	0.698914
Apr-21	1.911669	0.698917
May-21	1.911923	0.698919
Jun-21	1.912172	0.698921
Jul-21	1.912417	0.698923
Aug-21	1.912657	0.698924
Sep-21	1.912893	0.698926
Oct-21	1.913124	0.698928
Nov-21	1.913351	0.698929
Dec-21	1.913574	0.698931
Jan-22	1.913793	0.698932
Feb-22	1.914008	0.698934
Mar-22	1.914218	0.698935
Apr-22	1.914425	0.698936
May-22	1.914628	0.698937
Jun-22	1.914827	0.698939
Jul-22	1.915023	0.69894
Aug-22	1.915215	0.698941
Sep-22	1.915403	0.698942
Oct-22	1.915588	0.698942
Nov-22	1.915769	0.698943
Dec-22	1.915947	0.698944

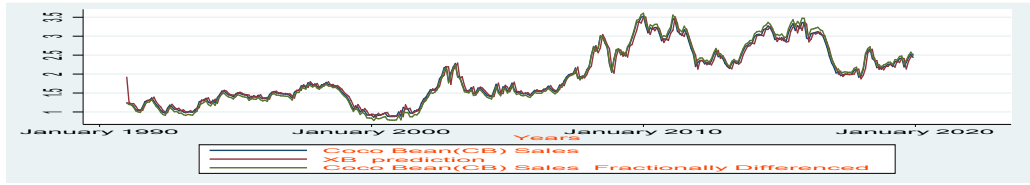


Fig. 14. Graphs by predictions appear to trail the data series

The above Fig. 14 shows that the in-sample predictions appear to trail the original series (Coco Bean Sales) well and that the fractionally differenced series looks much more like a stationary series than does the former (original) data series. Fig. 14 shows the original series (Coco Bean Sales), in-sample predictions and fractionally differenced Coco Bean Sales series. Also, Fig. 16 graph of impulse –response variable and impulse function and Figure fitting the ARMA into ARFIMA model. Figure fits the ARMA into ARFIMA model is examined to identify the deficiency associated with ARMA and ARIMA model respectively.

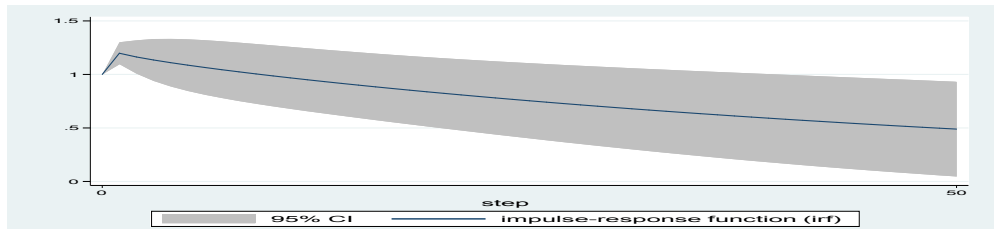


Fig. 15. Graphs by irfname, impuse variable, and response variable

The above Fig. 15 shows that a shock to a sale in Nigerian international market causes an initial spike to the commodity itself, after which the impact of the shock starts decaying slowly. This behavior is an evidence of long-memory processes

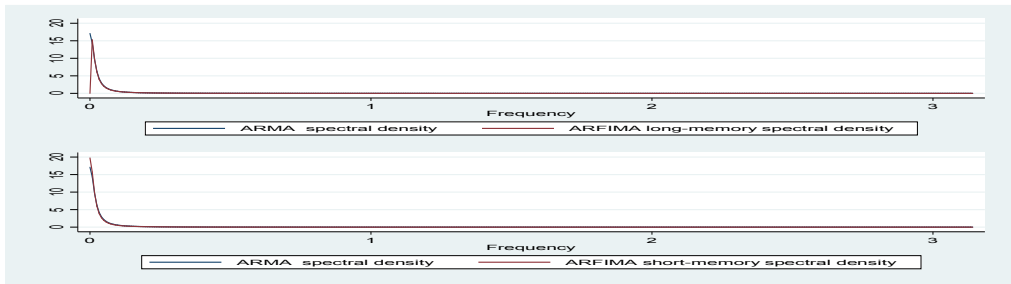


Fig. 16. Fitting the ARMA into ARFIMA model

4. Discussion

This study focuses on three important commodity prices representing Nigerian international markets. They include: Nigerian crude oil prices (Dollar Per Barrel), Coco Bean (Dollar) and exchange rate (USD/NGN). The data extracted for the study starts from 19th January, 1991 to 19th December, 2019. These three commodities future prices were expressed in Nigeria, Naira /US Dollar currencies and their return on prices i of commodity j , at definite time t in a continuous compounding basis were estimated in equation.

Figs. 1 – 3 represent the time plot of pattern of Nigerian international markets. These figures show an expected non-stationary process exists in price series. The series show continuous trending across the time of

the observations (data series) on vertical axis while the time is on the horizontal axis. Also, figs. 4-6 illustrate the time plot of the data series on returns on prices of Nigerian international markets (RIT) 100%. From physical observation, there is clustering effect in the return series, a period of high volatility followed by period of tranquility, such that Nigerian international markets future returns fluctuate in a range smaller than the normal distribution. This conformed to Roengchai [18] assertion that there are some circumstances where certain commodity future returns oscillate in a much wide scale that is allowed by a normal distribution. According to Mohammad Saeid [19] this process is important in order to recognize the distribution of the data and in a way prepare the financial series for modeling process.

Table 1 presents the summary descriptive statistics which include: mean, median, maximum, minimum, standard deviation, skewed test, excess kurtosis and Jarque-Bera test statistic for individual data indicators of the Nigerian international markets. The symbols used in describing them include: RCP (Returns on Crude Oil prices), RCB (Returns on Cocoa Bean Prices) and REXCH (Returns on Exchange Rate) respectively. The sample mean of all the indicators have positive signs which means they are positive mean reverting, but their corresponding standard deviation of the return series are much higher. Also, the skewed statistics are all positive except returns on crude oil prices which have negative sign (-0.679). This simply signifies that the series is skewed to the left (extreme loss) than right tail (extreme gain). A situation whereby an indicator is skewed to the left is one of the common characteristics of financial data series [20]. Also, the kurtosis statistics are 3.971, 3.891 and 9.618 respectively, suggesting the presence of heavy tail. The Jarque-Bera (J-B) test statistics are 40.320, 11.654 and 694.057 respectively and they are all statistically significant theory, indicating that the distribution of these indicators are not normally distributed. The causes of this could be attributed to the presence of extreme observations. Hence, the null hypothesis of normality is rejected while the alternative hypothesis that these indicators are not normally distributed is accepted. The results obtained here is in line with Roengchai [18] attempt “to model long memory volatility in Agricultural commodity futures returns”. In Roengchai [18], it was found that the normal distributions of the indicators under investigations have a small skewed statistic and high kurtosis suggesting that the distributions for the returns in prices were not normal. However, in a situation like this, Deebom and Essi [20] suggested that there is need to further use an alternative inferential statistic in order to eliminate the problems associated with non-normality distributions of the returns on prices of the commodity.

Table 3 shows the results of stationarity check in the series using the augmented Dickey Fuller Test (ADFT), Philip Perron Test (PPT) and the KPSS test respectively. The augmented Dickey Fuller test (ADFT) examines the null hypothesis that a time series data under investigation is stationary against the alternative hypothesis that the series non-stationary. The results obtain here revealed that since the probability value of the estimate is smaller than 0.05, level of significance, we concluded that the series is stationary at first difference. This result agrees with Omekara [15] and Kesemia [21,22] findings respectively in their studies carried out on forecasting liquidity ratio of commercial banks in Nigeria and the investigation of fractional ARIMA process and applications in modeling financial time series. In these studies, it was found that the series used in these studies were stationary at first-difference respectively. Similarly, their investigations were carried out using the Philip Perron test (PPT) and it was confirmed that the series are all stationary at first difference. In another development, these results were also checked by performing the KPSS test, the null hypothesis of the KPSS test confirmed that the series are stationary. All unit root tests were conducted with STATA version 15 and they were all tested at 5% level of significances.

Fig. 7-12 presents the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) of Nigerian international markets indicators. From visual examination, the autocorrelation decay rate shows that there is the presence of long memory in the monthly returns for Nigerian international markets indicators. Fig. 7, 9, and 11 shows the autocorrelation function of absolute Nigerian international markets monthly returns and the graphs show that the autocorrelation is persistent and significant at 26, 23 and 24 lags respectively. This is evidence of non stationary series, since they fall outside the 95% confidence interval while their partial autocorrelation function as shown in Fig. 8, 10 and 12 cut off at 2, 2, and 4 respectively. The Autocorrelation Function (ACF) tails off showing significant spikes at their respective lags while the partial autocorrelation functions showing a positive first lag and a set of exponential decays. This is an indication of an Autoregression AR(P) process, using the correlelogram. Otherwise if the first Lag of

the autocorrelation function of the difference shows a cut-off and/or the first lag is negative, then it is expected to add a moving average (MA) terms to the model. This result agrees with Kevin [23], Serlcarne & Veysel [24], Mukherjee & SarKar [25], and Brooks [26] on the methods to long memory, Christos [27] and Samet & Yanlin [28] findings respectively

However, from visual examination, both autoregressive (AR) and the moving average (MA) seem to move in the opposite direction. Although, Naveen [29] observes that traditional stationary ARMA (Autoregressive Moving Average) model has a short memory, as a result of its autocorrelation function decays exponentially. On the contrary, the ACF dies more slowly than theoretical autocorrelation at long lags. Naveen [29] further opined that the traditional stationary Autoregressive Moving Average (ARMA) models generally produce an excessive number of parameters for model estimation, specifically when the auto correlation decay date is slow. Based on our results in this case, reveal an auto correlations that exhibit a clear pattern of persistence and slow decay which is a typical case of a long-memory process as it was also confirmed in Kseniia [22]. In Kseniia [22] "Fractional ARIMA processes and applications in modeling financial time series", it was found that the autocorrelations of the data series under investigation exhibited a clear pattern of persistence and slow decay which is a clear case of a long-memory process [23].

Table 2 presents ARCH effect using the Ljung-Box (Q) statistics for the raw time series and the returns series. From the returns of the test statistics, the null hypothesis of white noise is rejected for the raw time series data while the alternative hypothesis is accepted. Also, in the case of the returns series the reverse is the case. This shows that the raw time series data are autocorrelated. This result confirmed with Kevin [23] findings in examining "an overview of the determinants of financial volatility: An explanation of measuring techniques". In Kevin [23] it was found that returns series do not show evidence of the presence of autocorrelation.

Table 4 shows the estimates for the test of the presence of long memory in Nigerian international markets. The study test the presence of Long memory in raw prices and in the returns on prices series, using Lo's rescaled statistics, Robinson test and GPH test. From the results obtained, it was confirmed that raw prices clearly show evidence of long memory using all the estimators. Therefore, in this case, we do reject the null hypothesis of no Long memory in Nigerian international markets while the alternative hypothesis was accepted. The implication on the economy is that there is display of a typical spectral shape, apparently diverging to infinity at zero frequency and declining uniformly in hyperbolic order which makes the economy unstable. For the return series, the results of all the tests provide evidence of no long memory. However, the results of long memory found in the raw prices series pointed out the suitability of a class of GARCH model which can capture long memory property in the volatility process. The result obtained under this investigation is not in line with Serlcarne and Veysel [25] findings. In Serlcarne and Veysel [25], "investigations of the value-at-risk predictions of precious metals with long memory volatility models", it was found that in investigating long memory property or precious metals using both GPH and GSP tests do not reject the null hypothesis of no long-memory for the returns on the prices of precious metals.

Table 6 shows that for sales, parameter (d) in crude oil prices $d \in (0.431, 0.497)$ for ARFIMA (1, d, 1) and $\epsilon \in (0.497, 0.431)$ for ARFIMA (2, d, 2) while for cocobeans have $(-0.021 < d < 0.497)$ for ARFIMA (1, d, 1) and ARFIMA (2, d, 2) respectively. Similarly, for exchange rate we have $d \in (0.075, 0.497)$ for ARFIMA (1, d, 1) and ARFIMA (2, d, 2) respectively. More specifically, for crude oil prices, empirical evidence shows that increase in Lag length may lead autocorrelations hyperbolic decay to zero. This is also true for others, except coco bean; when ARFIMA (2, d, 2) model is used, it shows evidence of an intermediate memory or anti persistence since autocorrelations is negative. The results obtain here conformed to Mukherjee and Sarkar [26], studied on "long memory in stock returns: Insights from the Indian markets". In Mukherjee and SarKar [26], three different tests were used to detect the presence of long memory in stock returns and they include: the rescale range analysis, the modified rescale range analysis and the whistle test. The result shows that the returns series do not exhibit any appreciable long memory process; the absolute and squared returns display long memory features. It was also revealed that the results of the findings are in agreement with the stylized facts observed in financial markets.

Table 7 shows the estimates of the application of long memory models in modeling sales and returns on prices in Nigerian international markets by using the FIGARCH- BBM and Chung model specifically the BFGS maximization method. The GARCH co-efficient (β) are all significant at 50% level of significance suggesting that the conditional variance depend upon its own lagged values for sales and returns on prices in Nigerian international markets, except the case of exchange rate returns on prices. Generally, the large (or small) sum of the ARCH and GARCH would infer that substantial positive or negative in sales or returns on prices in Nigeria international markets, in future forecast or the variance ,to remain high for a sustained period [27]. The long memory parameters values (d) are for sales are equal to 0.001, 0.000, 1.00, 0.961, 0.716 and 0.190 which shows evidence that the null hypothesis of no long memory should be rejected. These values are found significant at a 5% significance level.

Similarly, returns on prices were equal to 0.354, 0.45, 0.180, 0.45, and 0.282 and 0.450. This shows evidence that the null hypothesis or no long memory should be rejected at the 5% level of significance except the case of returns on prices of exchange rate are not significant. The implication for the significance or the d parameter is that any shock to volatility decay slowly at a hyperbolic rate item than the typical fast decay (exponential rate). The absence of long memory in the returns on prices of exchange rate contradicts Christos [27] findings in testing “Long memory in exchange rates: International evidence”. In Christos [27], it was found that lack of long memory in the daily returns of exchange rates supports the efficient market hypothesis (EMH). Although, the variations in the findings could be attributed to timing of the variable for example, the former study was on daily returns of exchange rates while the current study was on returns of exchange rates in Nigerian international markets[29]. Samet and Yanlin [28], investigates Long memory in volatilities of CDS spreads pieces of evidence from the emerging markets. From the results obtained using the FIGARCH model long-memory suggests that the Efficient Market Hypothesis (EMH) may not hold for the CDS spreads of those four countries under investigation. Similarly, this result agrees with Heitham [30] in evaluating the success of Fractionally Integrated-GARCH models on prediction stock market return volatility in gulf Arab stock markets. The purpose of the study was to evaluate the different Fractionally Integrated-GARCH Models which include; FIGARCH for BBM's, FIGARCH for Chung, FIEGARCH, FIAPARCH for BBM's, FIAPARCH for Chung, and HYGARCH. The results show that FIGARCH BBM is the best fitted model for the data extracted from UAE, KSA, and Bahrain.

Table 9 contain the result for test for the presence of ARCH effect which shows that the decision to accept the hypothesis should be upheld while the alternative hypothesis will be rejected since the estimated probability value is greater than the standard probability value of 0.005. The implication for this is that there is absence of ARCH effect. Similarly, we accept the null hypothesis since the calculated probability value is greater than 5% level of significance. Conversely, we will fail to accept the alternative hypothesis

Fig. 14, shows that the theoretical lie straight which reveals that the residual follow a standardized order of a normal distribution and this is an example of a normal Q-Q plot since both sets of quantiles lies straight on each other and this confirms truly that they come from normal distribution.

Table 10 shows performance evaluation of ARFIMA (1,-0.021,1)model. After checking the adequacy of the model selected, the result reveals the predicted values of the estimated model. The implication of this is that the estimated ARFIMA (1,-0.021, 1) model is better fitted to the application of Long memory model in Nigerian international markets. This result is similar to Omekara (15) and Kesemia [22] findings respectively. In these studies, carried out by Omekara [15] and Kesemia [22] in forecasting liquidity ratio of commercial banks in Nigeria and in the investigation of fractional ARIMA process and applications in modeling financial time series, it was found that ARFIMA (5, 0.12, 1) model better than other models.

Fig. 14 shows that the in-sample predictions appear to trail the original series (Coco Bean Sales) well and that the fractionally differenced series looks much more like a stationary series than does the formal (original) data series. Similarly, Fig. 16 shows that a shock to coco bean sale causes an initial spike to the commodity itself, after which the impact of the shock starts decaying slowly. This behavior is an evidence of Long memory processes.

Fig. 16 contains a plot of the spectral densities discussed by both ARMA and the long-run ARFIMA parameter estimates. This was plotted to examine the deficiency associated with ARMA process that led to development of the ARFIMA model as generalizations of the ARIMA models. The result obtained for fitting the short memory into long memory model conformed to Granger and Joyeux's [9] assertion. In Granger and Joyeux's [9], it was asserted that the two models imply different spectral densities for frequencies close to 0 when $d > 0$. When $d > 0$, the spectral density implied by the long memory estimates diverges to infinity, whereas the spectral density implied by the ARMA estimates remains finite at frequency 0 for stable ARMA processes. This difference reveals the ability of ARFIMA models to capture long-run effects which ARMA models only capture as the parameters approach specifically for an unstable model. The second graph in figure shows a plot of the spectral densities implied by the ARMA parameter estimates and by the long memory (short-run ARFIMA) parameter estimates, which are the short memory models (ARMA) parameters for the fractionally differenced process. However, comparing the two graphs shows the ability of parameters of long memory (short-run ARFIMA) to capture both low and high-frequency components in the fractionally differenced series. In contrast, parameters of short memory models (ARMA) can only capture low-frequency components in the fractionally integrated series. All these agree with Palma [30] in modeling Long-Memory Time Series: theory and Methods.

4.1 Summary of Major Findings

The persistence of shocks in Nigerian international Markets is shown in Figs. 4-6 which are illustrated using the time plot of the data series on returns on prices of Nigerian international markets (RIT) 100%. From physical observation, these are clustering effects in the return series, a period of high volatility followed by a period of tranquility, such that Nigerian international markets future returns fluctuate in a range smaller than the normal distribution. Also, in Fig. 16 there is the presence of persistence shocks in International Markets (cocoa bean sales) which is caused by initial spike in the commodity itself, after which the impact of the shock starts decaying slowly. This behavior is evidence of long-memory processes. Also, Using Lo's rescaled statistics, Robinson test, and GPH test, it was revealed that there is the presence of long memory in the sales but not too visible in the returns on prices in Nigerian international markets. Also, to develop a Long-range dependence in Nigerian international Markets is done using fractionally integrated volatility models as shown in Equation 3.3 and 3.4 in chapter 4 of this study. Although, from the results it is found that returns series do not show much evidence of the presence of autocorrelation. Based on the results of the data analysis, the appropriate long memory model for Nigerian international Markets between 1991 to 2020 can be written as thus:

$$\text{ARFIMA}(1, -0.021, 1) = \begin{matrix} 0.984(D)(1-D)^{-0.021}(1_t - \mu) = 0.236(D)\varepsilon_t \\ (0.000) \qquad \qquad \qquad (0.000) \end{matrix} \quad 4.1$$

$$\text{AIC} = -531.204 \quad \text{BIC} = -511.943$$

Some of the advantages in using long memory models in the modeling of the returns in Nigerian International Markets over the use of short memory models is that the ARIMA models only capture the short-range dependence property which is part of the normal conventional integer-order models, whereas the ARFIMA model gives a better fit and result when dealing with the data which possess the long-range dependence property. Empirically, From the selected models it was confirmed that ARFIMA (1,d,1) shows evidence of better performance than any other long memory models. In terms of performance comparison, the results obtained here conformed with Li et al[31] findings in their study on the use of the ARFIMA model to analyzing and predicting the future levels of the elevation of Great Salt Lake data. The results revealed that the predicted results of long models have better performance compared to the conventional ARMA models. In another development, Li et al[31] examined four models for the Great Salt Lake water level forecasting: ARMA, ARFIMA, Generalized Autoregressive Conditional Heteroskedasticity (GARCH), and Fractional Integral Autoregressive Conditional Heteroskedasticity (FIGARCH). They found that FIGARCH offers the best performance, indicating that conditional heteroscedasticity should be included in

time series with high volatility. Also, Sheng and Chen [32], proposed a new ARFIMA model with stable innovations to analyze the Great Salt Lake data and predicted future levels. They also compared accuracy with previously published results. Also, the implemented methods are illustrated with applications to some numerical examples and a tree ring database [33]. Also, in the application of both univariate and multivariate trend-stationary ARFIMA models generated a long memory autocorrelated process around a deterministic time trend. Baillie and Chung [34] found remarkable success in representing the annual temperature and width of tree ring time-series data. Based on performance evaluation using the Aikai information criteria as one of the several types of model selection criteria, the ARFIMA (1,-0.021, 1) model was found to be the best-fitted model. Therefore, an ARFIMA (1,-0.021,1) model should be considered appropriate in testing the application of long memory models in modeling sales and returns on prices in any other international markets. It was confirmed that the Efficient Market Hypothesis (EMH) may not hold for Nigerian International markets. For two years (January, 20 to Dec-22) step ahead forecast, the predicated value for Cocoa Bean Sale using the ARFIMA (1,-0.021,1) falls between the range of 1.907247 to 1.915947.

5 Conclusion

This study mainly focused on the application of long-memory models in sales and returns on prices in Nigerian international markets. To determine the presence of long-memory in returns and volatility, we utilized two traditional approaches to long memory modeling such as the classical stationary time series and fractionally integrated model before estimating the ARFIMA and FIGARCH models. The results obtained from the empirical findings suggest that the series (sales and returns on price) appears to be leptokurtic. It exhibits heavy-tail behavior and the returns series confirming that the indicators of Nigerian international markets are autocorrelated. The results of ARFIMA and FIGARCH models show that there is the presence of long memory in Nigerian international markets (sales). It also shows that sales and returns on prices are predictable. This supports the concept of a long-memory process. In conclusion, from the results obtained the study conducted a forecast of the indicators of Nigerian International Markets using ARFIMA (1,-0.021,1). The autocorrelation function of the Nigerian International Markets data showed persistence characteristic which is one of the features of a long memory process. This led to fitting a suitable ARFIMA (1,-0.021,1) model. This indicates that shocks in Nigerian International Markets persist over a long period and so, there is a need for government and policymakers to pursue policies that will strongly stabilize and regulate the activities of investors in Nigerian International Markets. Besides, since Nigeria is an exporting and importing economy, policies that will ensure a good net export status for the economies should be pursued vigorously. The study suggested that in the design for asset allocation strategies, government, policymakers, investors, market experts should consider oil price and agricultural commodity price volatility as a key source of inevitable risk.

Competing Interests

Authors have declared that no competing interests exist.

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