Finite Euclidean Geometry Approach for Constructing Balanced Incomplete Block Design (BIBD)

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Authors' contributions

This work was carried out in collaboration among all authors. Author UPA designed the study and establishing the technique used in this work. Author UPA and SSA construct BIBDs using EG(N, s) approach. Author TAU wrote the literature of the work. Author OEN did the geometrical construction of BIBD. All the authors read and approved the final manuscript.

Original Research Article

Abstract

In block design, construction of Balanced Incomplete Block Design (BIBD) remained an unsolved problem in combinatorial design; also various construction techniques have been introduced to build the elements of BIBDs for specific parameters; no general method has been proposed to find a suitable structure for BIBDs. This paper aim at employing Finite Euclidean Geometry FEG (N,s) of N – dimensional space to construct balanced incomplete block design (BIBD). Also geometrical construction of FEG (2,2) BIBDs has been made. The results show that this technique proved a better method for constructing BIBD than other methods in terms of estimation of parameters to build the design structure.

1 Introduction

Block designs is originated from the statistical framework of design of experiments. The generation of block designs is a well-known combinatorial problem, which is very hard to solve [1]. This design has wider

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applications from many areas, including experimental design, finite geometry, physical chemistry, software testing, cryptography and algebraic units [2]. A block design B has a non-empty domain $X = \{x_1, x_2, \ldots, x_v\}$ whose elements are sometimes called varieties and a non-empty collection of subsets of X called blocks. A block design is a simple design if no two blocks are identical. The origins of incomplete block designs dated back to [3] who introduced the concept of balanced incomplete block designs and their analysis using both intra- block and inter - block information. When the number of varieties to be compared is large, a large number of blocks to accommodate all the varieties will be needed for the experiments. This requires more experimental material and so the cost of experimentation becomes very high. The completely randomized design and randomized block design may not be suitable in such situations because they will require large number of experimental units to accommodate all the treatments. When sufficient numbers of homogeneous experimental units are not available to accommodate all the treatments in a block, then incomplete block designs can be used [4]. A block design is incomplete if the number of varieties is greater than the block size ($k < v$), then the design is incomplete block design. Narelle &Deborah [5] investigated the best balanced incomplete block design used to estimate the parameter rates for a smaller variance than estimates obtained using the similar design.

The idea of balancing a design with incomplete blocks was introduced by [6] in his theoretical study of the design of experiments in agriculture. A balanced design is complete if $k = v$, so that each block contains all of X. A balanced incomplete block design (BIBD) is an incomplete block design which consists of a set of v points that is divided into b subsets in such a way that each point in v is contained in r different subsets and any couple of points in v is contained in $\lambda < b$ subsets with $k < v$ points in each subset. A BIBD is specified by five parameters (v, b, r, k, λ). The five parameters defined as (v, b, r, k, λ) - BIBD are related and satisfy the following two relations: $bk = vr$ and $\lambda(v-1) = r(k-1)$. Alabi, [7] construct balanced incomplete block designs using method of lattice or orthogonal designs of series I and II due to Yates Algorithm and Khare and Federer. The generation of block designs remains an unsolved problem in combinatorial mathematics [8]. However, a balanced incomplete block design is an incomplete block design in which any two varieties appear together an equal number of times. This design can be constructed by taking $\begin{pmatrix} v \\ k \end{pmatrix}$ blocks and assigning a different combination of varieties to each block [9]. A standard way of representing a BIBD is in terms of its incidence matrix $M \equiv \{m_{ij}\}_{v \times b}$, which is a $v \times b$ binary matrix with exactly r ones per row, k ones per column, a scalar product of λ between any pair of distinct rows, and where $m_{ij} \in \{0,1\}$ is equal to 1 if the ith object is contained in the jth block, and 0 otherwise. In this context, m_{ij} represents the incidence of object i in block j of M. Janardan [10] used Galois field approach to construct BIBD through constructing Mutually Orthogonal Latin Square Design (MOLSD). Very – large – scale integration (VLSI) design methodology is used to fit a circuit for balanced incomplete block design (BIBD) and this scheme generates and simulates a BIBD pattern proposed by [11]. The construction of such design depends on the actual arrangement of the treatments into blocks and this problem is handled in geometry which advanced the purpose of this paper.

2 Basic Concept of Balanced Incomplete Block Design

An incomplete block design (k < t) is said to be a balanced incomplete block design (t, b, r, k, λ) or (t, k, λ) - BIBD in a design (X, A) if the following postulates are satisfied;

(i) $|X| = t$
(ii)   Each block contains exactly k points
(iii)   Every pair of distinct contained points exactly λ blocks

The notations $(t, b, r, k$ *and* $\lambda)$ are called the parameters of the balanced incomplete block (BIB) design. The parameter t is the number of variety, b is the number of block, r is the number of replicate observation per variety, k is the number of observation per block and λ is the number of times any two varieties occur together in a block. The following relations exist among the parameters $(t, b, r, k$ *and* $\lambda)$. The necessary conditions for the existence of (t, b, r, k, λ) - design are stated below:

(i) $rt = bk$ (1)

(ii) $r(k-1) = \lambda(t-1)$ (2)

(iii) $r > \lambda$ (3)

(iv) $b = t$ (4)

The relationship (1) follows from the fact that the total number of observation for the balanced incomplete block design is:

$$N = \sum_i \left(\sum_j n_{ij} \right) = \sum_j \left(\sum_i n_{ij} \right)$$ (5)

Proposition 1: For every non − empty (t, b, r, k, λ) - BIBD;

(a) $\lambda > 1$
(b) $k < t$

Proof:

(a) Let b be the block in (t, b, r, k, λ) - BIBD with at least two elements, then $\exists$ at least one block and some pair has at least once occurrence. Therefore, all pairs occur equally, it follows that $\lambda \geq 1$.

(b) Let X be a domain and k is the size, then $b \subset X$. This follows that the block size cannot exceed the size of the domain. Thus, $k \leq t$. Since a BIBD is incomplete, it follows that $k < t$.

2.1 Proposition

The parameters of a BIBD on $X = \{x_1, x_2, \ldots, x_t\}$ satisfy the following two conditions:

(i) $bk = rt$
(ii) $\lambda(t-1) = r(k-1)$

Proof:

(i) First, consider the $t \times b$ incidence matrix; $I_{ij} = \begin{cases} 1 \ if \ x_i \in B_j \\ 0 \ otherwise \end{cases}$

$$I = \begin{array}{c|cccccc} & B_1, & . & . & . & . & , B_b \\ \hline x_1 & I_{1,1} & . & . & . & . & I_{1,b} \\ . & . & . & . & . & . & . \\ . & . & & & & & . \\ . & . & & & & & . \\ x_t & I_{t,1} & . & . & . & . & L_{t,b} \end{array}$$

Fig. 1. Incidence matrix showing the numbers of varieties (t) arranged in blocks (b) of k units each being replicated (r) times

There are t rows each with row – sum r, and there are b columns each with column – sum k. therefore, $bk = rt$

(ii) Consider $\binom{t}{2} \times b$ pair incidence matrix

$$I' = \begin{array}{c|cccccc} & B_1, & . & . & . & . & , B_b \\ \hline x_1 x_2 & I'_{12,1} & . & . & . & . & I_{12,b} \\ . & . & . & . & . & . & . \\ . & . & & & & & . \\ . & . & & & & . & \\ x_{t-1} x_t & I'_{(t-1),t,1} & . & . & . & . & L_{(t-1),t,b} \end{array}$$

With $I'_{ij,l} = \begin{cases} 1 & if \ x_i x_j \in B_l \\ 0 & otherwise \end{cases}$

There are $\binom{t}{2}$ rows each with row – sum λ and there are b columns each with column –sum $\binom{k}{2}$.

Therefore, $\lambda \binom{t}{2} = b \binom{k}{2}$

$$\lambda \left(\frac{t!}{t!(t-2)!} \right) = b \left(\frac{k!}{k!(k-2)!} \right)$$

$\Rightarrow \lambda t (t-1) = bk (k-1)$

$\Rightarrow \lambda t (t-1) = tr (k-1)$

$\Rightarrow \lambda (t-1) = r (k-1)$, since $tr = bk$

2.2 Proposition

In a (t,k,λ) - BIBD, every point occurs in exactly $r = \dfrac{\lambda(t-1)}{k-1}$ blocks.

Proof:

Let (E,ψ) be a (t,k,λ) - BIBD. Let $v \in E$ and r_v be the number of blocks containing E. Define a set $F = \{(y,\psi) : y \in E,\ y \neq v,\ W \in \psi, (v,y)\} \subseteq W$

We can compute F in two ways;

There are $(t-1)$ to choose $y \neq v \in E$, and for each y, there is exactly λ blocks such that $\{v, y\} \subseteq W$ $|F| = \lambda(t-1)$

Secondly, there are r_v block $v \in W$. For each W, there are $k-1$ ways to choose $y \in W$ and $y \neq v$. Hence $|F| = r_v(k-1)$
Combining these two equations of $|F|$, we have;

$$\lambda(t-1) = r_v(k-1)$$

Hence $r_v = \dfrac{\lambda(t-1)}{(k-1)}$

The r_v is independent of v and any element is combine in exactly r number of blocks.

Therefore, $r = \dfrac{\lambda(t-1)}{k-1}$ as required.

2.3 Proposition

For every non – void design, a (t,k,λ) - BIBD has exactly $b = \dfrac{tr}{k} = \dfrac{\lambda(t^2-t)}{k^2-k}$ blocks.

Proof:

Let (Y,C) be a (t,k,λ) - BIBD and let $b = |C|$. Define a set $S = \{(y,C) : y \in Y,\ A \in C,\ y \in A\}$, then $|S|$ can be computed in two ways.

Firstly, there are t ways of choosing an element $y \in Y$. For each y, there are exactly r blocks such that $y \in A$. Hence $|S| = tr$

Secondly, there are b blocks $A \in C$. For each block A, there are k ways to choose $y \in A$. Hence $|S| = bk$.

Combining the two equations, we get;

$$bk = tr \Rightarrow b = \frac{tr}{k}$$

Substituting the value of r from the above theorem (3.1) in $b = \dfrac{tr}{k}$ yields

$$b = \frac{t\,\lambda(t-1)}{k\,(k-1)} = \frac{\lambda(t^2-t)}{k^2-k} \text{ as required}$$

3 Incidence Matrix

Let (X,ψ) be a design where $X = (x_1,x_2, ...,x_t)$ and $\psi = \{\psi_1, \psi_2,. . . . .,\psi_b\}$. The incidence matrix of (X,ψ) is the $t \times b_{0-1}$ matrix $M = (m_{ij})$ defined as:

$$m_{ij} = \begin{cases} 1 \text{ if } x_i \in \psi_j \\ 0 \text{ if } x_i \notin \psi_j \end{cases}$$

(6)

From the definition and the properties of a BIBD, it is clear that the incidence matrix M of a (t,b,r,k,λ)-BIBD satisfies the following properties

(1)   Every column of M contain exactly K'1's
(2)   Every row of M contain exactly r'1's
(3)   Two distinct rows of M contain both 1's in exactly λ columns

4 Finite Geometry Design

Combinatorial designs on a finite set of elements are known as a finite geometry. The elements of their domains are points of the geometry and their different subsets are lines of the geometry. The points of the geometries are known as varieties and the lines are known as blocks. The two standard axioms below are generally used in geometries:

A1: Two distinct points contained at most on a line.
A2: Two distinct lines intersect at most one point

The incidence matrix of geometry $<X,L>$ with v points: $X = \{x_1,x_2,\ldots,x_v\}$ and L lines: $L = \{l_1,l_2, ...,l_l\}$ is the $v \times l$ matrix

$$M_{<X,L>(i,j)} = \begin{cases} 1 \text{ if } x_i \in L_j \\ 0 \text{ otherwise} \end{cases}$$

(7)

Geometry is commonly specified by its incidence matrix. The dual of a geometry $<X,L>$ is the geometry $<X^*,L^*>$ with $X^* = L$ and $L^* = X$, whose incidence matrix is the transpose of the incidence matrix of $<X,L>$

4.1 Finite Euclidean Geometry Design

The set of all points of Projective geometry, $PG(N,p^n)$ whose first coordinate x_0 is not zero is known as Euclidean geometry of N – dimension denoted by $EG(N,p^n)$. In this geometry, the points are given by the

ordered set $(x_1, x_2, \ldots, x_N)$ where the $x_i's$ are the elements of Galois field, $GF(p^n)$. The number of points in $EG(N, p^n)$ is S^N, where $S = p^n$. The k – flat of $EG(N, p^n)$ is form from all the points which satisfy a set of consistent and independent linear equations represented by the equations (8) below.

$$
\left.
\begin{array}{l}
a_{10} \quad + \quad a_{11}x_1 \quad + \quad a_{12}x_1 \quad + \quad \ldots \quad + \quad a_{1N}x_N \quad = 0 \\
a_{20} \quad + \quad a_{21}x_1 \quad + \quad a_{22}x_2 \quad + \quad \ldots \quad + \quad a_{2n}x_N \quad = 0 \\
\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
a_{N-m,0} \quad + \quad a_{N-m,1}x_1 \quad + \quad a_{N-m,2}x_2 \quad + \quad \ldots \quad + \quad a_{N-m,N}x_N \quad = \quad 0
\end{array}
\right\}
\tag{8}
$$

The number of k – flat in $EG(N, p^n)$ is:

$$
Q(N, m, s) - Q(N-1, m, s) = S^{N-m} Q(N-1, m-1, s) \tag{9}
$$

$$
Q(N, m, s) = S^{N-m} Q(N-1, m-1, s) + Q(N-1, m, s) \tag{10}
$$

To every point of $EG(N, p^n)$ there is a corresponding variety. To every k – flat there exist a corresponding block containing all those varieties whose corresponding points form the m – flat. Then the design obtained from the flat is shown below:

$$
v = Q(N, 0, s) - Q(N-1, 0, s) = S^N \tag{11}
$$

$$
b = Q(N, m, s) - Q(N-1, m, s) = S^{N-m} Q(N-1, m-1, s) \tag{12}
$$

$$
r = Q(N-1, m-1, s) \tag{13}
$$

$$
k = Q(N, 0, s) - Q(m-1, 0, s) = S^m \tag{14}
$$

$$
\lambda = Q(N-2, m-2, s) \tag{15}
$$

If M and m are taken as fixed, then all the designs by (11 - 15) for various values of $s = p^n$ may be said to belong to the series $\left(E_N^m \right)$.

4.2 Implementation

Construction 1:

Consider a design $\left(E_2^2 \right)$ series for which S = 2. Construct BIBD of $EG(2,2)$.

In this design, N = 2, $S = p^n = 2$ the number of points in $EG(2,2) = S^N$. The parameters (t, b, r, k, λ) - BIBD are obtained as;

$$
t = \Omega(N, 0, s) - \Omega(N-1, 0, 1) = 4
$$

$$
b = \Omega(N, m, s) - \Omega(N-1, m, s) = 6
$$

$$r = \Omega(N-1, m-1, s) = 3$$

$$k = \Omega(m, 0, s) - \Omega(m-1, 0, s) = 2$$

$$\lambda = \Omega(N-2, m-2, s) = 1$$

In $EG(2,2)$ every point has coordinate of the form x_1, x_2. The blocks are obtained from the equations below;

$$x_i = 0 \, or \, 1 \; (i = 1, 2)$$

$$x_i + x_j = 0 \, or \, 1 \; (i, j = 1, 2; i \neq j)$$

$$x_1 + x_2 = 0 \, or \, 1$$

The blocks are generated from the equation $2x_1 + x_2$

$$
\begin{array}{llll}
x_1 = 0: & x_1 = x_2 = 0, & x_2 \\
x_2 = 0: & x_1 = x_2 = 0, & x_1 \\
x_1 + x_2 = 0: & x_1 = x_2 = 0, & x_1 + x_2 \\
x_1 = 1: & x_1, & x_1 + x_2 \\
x_2 = 1: & x_2, & x_1 + x_2 \\
x_1 + x_2 = 1: & x_2, & x_1
\end{array}
$$

Table 1. Transforming the co-ordinates, each point has a unique number between 1 and 6. The number corresponds to x_1, x_2 is in line with the equation $2x_1 + x_2$. The solution can be written in a tabular form as;

Points	Co – ordinates		Blocks	
	x_1	x_2	$x_i = 0$	$x_i = 1$
0				
1	0	1	$x_1 = 0: 0 \, 1$	$x_1 = 1: 2 \; 3$
2	1	0	$x_2 = 0: 0 \, 2$	$x_2 = 1: 1 \, 3$
3	1	1	$x_1 + x_2 = 0: 0 \, 3$	$x_1 + x_2 = 1: 1 \, 2$

Therefore $EG(2,2)$ form a design below;

$$X = \{0, \; 1, \; 2, \; 3\}$$

$$B = \{(01), (02), (23), (13), (03), (12)\}$$

Hence $EG(2,2)$ is $(4, 6, 3, 2, 1)$ - BIBD.

Construction II

Consider a design $\left(E_3^2\right)$ series for which S = 2. Construct BIBD of $EG(3,2)$.

In this design, N = 3, $S = p^n = 2$. From equations (11 – 15), the parameters (t, b, r, k, λ) - BIBD are obtained as;

$t = \Omega(N,0,s) - \Omega(N-1,s) = 8$

$b = \Omega(N,m,s) - \Omega(N-1,m,s) = 14$

$r = \Omega(N-1,m-1,s) = 7$

$k = \Omega(m,0,s) - \Omega(m-1,0,s) = 4$

$\lambda = \Omega(N-2,m-2,s) = 3$

In $EG(3,2)$ every point has coordinate of the form x_1, x_2, x_3. The blocks are obtained from the equations below;

$x_i = 0 \, or \, 1 \ (i = 1,2,3)$

$x_i + x_j = 0 \, or \, 1 \ (i, j = 1,2,3; i \neq j)$

$x_1 + x_2 + x_3 = 0 \, or \, 1$

The blocks are generated from the equation $4x_1 + 2x_2 + x_3$

$x_1 = 0: x_1 = x_2 = x_3 = 0, \quad x_3, \quad x_2, \quad x_2 + x_3$

$x_2 = 0: x_1 = x_2 = x_3 = 0, \quad x_3, \quad x_1, \quad x_1 + x_3$

$x_3 = 0: x_1 = x_2 = x_3 = 0, \quad x_2, \quad x_1, \quad x_1 + x_2$

$x_1 = 1: x_1, \quad x_1 + x_3, \quad x_1 + x_2, \quad x_1 + x_2 + x_3$

$x_2 = 1: x_2, \quad x_2 + x_3, \quad x_1 + x_2, \quad x_1 + x_2 + x_3$

$x_3 = 1: x_3, \quad x_2 + x_3, \quad x_1 + x_3, \quad x_1 + x_2 + x_3$

$x_1 + x_2 = 0: x_1 = x_2 = x_3 = 0, \quad x_3, \quad x_1 + x_2, \quad x_1 + x_2 + x_3$

$x_1 + x_3 = 0: x_1 = x_2 = x_3 = 0, \quad x_2, \quad x_1 + x_3, \quad x_1 + x_2 + x_3$

$x_2 + x_3 = 0: x_1 = x_2 = x_3 = 0, \quad x_1, \quad x_2 + x_3, \quad x_1 + x_2 + x_3$

$x_1 + x_2 = 1: \quad x_2, \quad x_2 + x_3, \quad x_1, \quad x_1 + x_3$

$x_1 + x_3 = 1: \quad x_3, \quad x_2 + x_3, \quad x_1, \quad x_1 + x_2$

$x_2 + x_3 = 1: \quad x_3, \quad x_1 + x_3, \quad x_2, \quad x_1 + x_2$

$x_1 + x_2 + x_3 = 0: x_1 = x_2 = x_3 = 0, \quad x_2 + x_3, \quad x_1 + x_3, \quad x_1 + x_2$

$$x_1 + x_2 + x_3 = 1: \quad x_3, \quad x_2, \quad x_1, \quad x_1 + x_2 + x_3$$

Table 2. Transferring the co-ordinates, each point has a unique number between 1 and 8. The number corresponds to x_1, x_2, x_3 is in harmony with the equation $4x_1 + 2x_2 + x_3$. The solution can be written in a tabular form as;

Points	Co – ordinates			Blocks	
0	x_1	x_2	x_3	$x_1 = 0$	$x_1 = 1$
1	0	0	1	$x_1 = 0 : 0\ 1\ 2\ 3$	$x_1 = 1 : 4\ 5\ 6\ 7$
2	0	1	0	$x_2 = 0 : 0\ 1\ 4\ 5$	$x_2 = 1 : 2\ 3\ 6\ 7$
3	0	1	1	$x_1 = 0 : 0\ 2\ 4\ 6$	$x_3 = 1 : 1\ 3\ 5\ 7$
4	1	0	0	$x_1 + x_2 = 0 : 0\ 1\ 6\ 7$	$x_1 + x_2 = 1 : 2\ 3\ 4\ 5$
5	1	0	1	$x_1 + x_3 = 0 : 0\ 2\ 5\ 7$	$x_1 + x_3 = 1 : 1\ 3\ 4\ 6$
6	1	1	0	$x_2 + x_3 = 0 : 0\ 4\ 3\ 7$	$x_2 + x_3 = 1 : 1\ 5\ 2\ 6$
7	1	1	1	$x_1 + x_2 + x_3 = 0 : 0\ 3\ 5\ 6$	$x_1 + x_2 + x_3 = 1 : 1\ 2\ 4\ 7$

Therefore $EG(3,2)$ form a design below

$$X = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7\}$$

$$B = \begin{cases} (0123),\ (0145),\ (0246),\ (0167),\ (0257),\ (0437),\ (0356),\ (4567),\ (2367), \\ (1367),\ (2345),\ (1346),\ (1526),\ (1247) \end{cases}$$

Hence $EG(3,2)$ is $(8, 14, 7, 4, 3)$ - BIBD.

5 Geometrical Construction of EG (2, 2) – Design

The incident matrix of EG (2,2) is given as;

$$M_{X,L} = \begin{array}{c} \\ x_0 \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{pmatrix} l_1 & l_2 & l_3 & l_4 & l_5 & l_6 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

The above incidence matrix gives rise to the construction of geometry shown (Fig 1) below. The geometry is an illustration of balanced incomplete block design with points and lines. This is a clear picture that balanced incomplete block design can be interpreted geometrically.
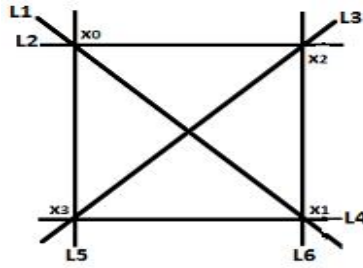
Fig. 1. A geometry with 4 points and 6 lines

The dual geometry of (X, L) is the geometry (X*, L*) which is the transpose of the incidence matrix (X, L).

$$
M^*_{X,L} = \begin{array}{c} \\ l^*_1 \\ l^*_2 \\ l^*_3 \\ l^*_4 \\ l^*_5 \\ l^*_6 \end{array}
\begin{array}{cccc}
x^*_0 & x^*_1 & x^*_2 & x^*_3 \\
\end{array}
\left(
\begin{array}{cccc}
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 \\
\end{array}
\right)
$$

The dual geometry can also be used to explain the structure of balanced incomplete block design based on points and lines.
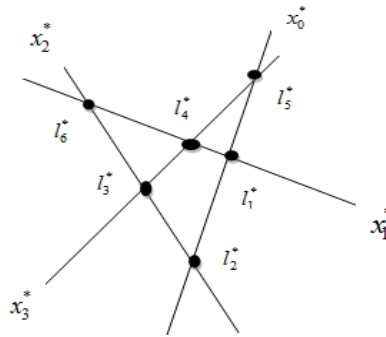

Fig. 2. Dual geometry design with 6 points and 4 lines

Hence, Figs. 1 and 2, shows the geometrical representation of (4, 6, 3, 2, 1) – BIBD

Geometrical construction of FEG (3, 2) for BIBD is not the scope of this paper; hence give room for further study.

6 Discussion

For a BIBD to exist, λ must be positive integers. The proposition (1) defined the condition that λ must be strictly greater than one and the block size must less than the number of varieties. Propositions (2), (3) and (4) show the relationship which make design a BIBD. Estimation of parameters of BIBD is obtained using

equation (11) – (15). Two designs of EG (2, 2) and EG (3, 2) were constructed to obtain different BIBDs. Geometrical construction of (4, 6, 3, 2, 1) – BIBD were established in Fig. 1, that shows 4 points and 6 lines using incidence matrix. The duality of the same design is constructed in Fig. 2, showing the geometrical structure with 6 points and 4 lines. These figures interpret BIBD in geometrical manner.

7 Conclusion

Every point in $EG(N,s)$, has the co − ordinate of the form $x_1 x_2 x_3, \ldots, x_p$. In each co − ordinate, two elements either 0 or 1 are considered to obtain the required number of blocks. The number of points in $EG(N,s)$ is obtained using the expression S^N. For $EG(2,2)$ and $EG(3,2)$, every point has the co − ordinate of the form $x_1 x_2$ and $x_1 x_2 x_3$. The blocks for the two geometries $EG(2,2)$ and $EG(3,2)$ are generated from the equation $2x_1 + x_2$ and $4x_1 + 2x_2 + x_3$ where their first co − ordinate is not zero (0). The designs (E_2^2) and (E_3^2) series generate (4, 6, 3, 2, 1) and (8, 14, 7, 4, 3) – BIBDs. It is observed that this technique is easy to estimate the parameters of BIBD and give a suitable construction of balanced incomplete block design (BIBD) irrespective of the block size and number of varieties.

Competing Interests

Authors have declared that no competing interests exist.

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