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Achieving System Reliability with the Fewest Identical Components

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

Generally, redundancy allocation problems are NP-hard. This paper presents an explicit polynomially bounded algorithm for a special class of redundancy allocation models.

Keywords: NP-hard; polynomially bounded algorithms; redundancy allocation; discrete optimization models.

2010 Mathematics Subject Classification: 90C99, 90C10, 90C30.

1 Introduction

Chern [1] studied the complexity of optimization models for redundancy allocation. Although his main result shows that, generally, redundancy allocation is NP-hard, he identified special cases that can be solved with polynomially bounded algorithms. This work deals with two of these cases:

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$$
\begin{array}{ll}
Model\,1 & Model\,2 \qquad (1.1)\\
\text{Max}\prod_{i=1}^{n}(1-\rho^{x_i}) & \text{Min}\sum_{i=1}^{n} x_i \qquad (1.2)\\
\text{Subject to}\sum_{i=1}^{n} x_i \le z & \text{Subject to}\prod_{i=1}^{n}(1-\rho^{x_i}) \ge R \qquad (1.3)\\
x_i \text{ integral, } x_i \ge 1 & x_i \text{ integral, } x_i \ge 1 \qquad (1.4)
\end{array}
$$

Soltani [2] has surveyed the literature on redundancy allocation and classified a wide range of models, of which ours are basic examples.

In the context of redundancy allocation, a system is modeled as a series of independent subsystems, and the parameter, n, represents the number of these subsystems. Initially, all components operate simultaneously and then maintain the function of the subsystem until the last component has failed. Failures of these components are treated as probabilistically independent events. Elsayed [3], and Kapur and Pecht [4] developed functions for the reliability of complex systems, including redundancy allocation problems modeled in model 2.

For the simplified models considered here, it is further assumed that all components of all subsystems are identical; the parameter, ρ, represents their common failure probability. An assignment of components to subsystems can be represented by an n-vector,
$x = (x_1, x_2, \cdots, x_n)$, of positive integers. Each coordinate shows the number of components allocated to the corresponding subsystem and the product $\prod_{i=1}^{n}(1-\rho^{x_i})$ is the reliability of the system. Throughout, it will be convenient to use the notation $R(x) = \prod_{i=1}^{n}(1-\rho^{x_i})$.

For Model 1, the final parameter, z, is an integer that represents the total number of components available for allocation among the n subsystems. Since each subsystem requires at least one component, $z \ge n$. An optimal solution shows how to configure the available components to maximize the reliability of the entire system. For Model 2, the final parameter is a value R, with $0 < R < 1$, which can be interpreted as a reliability requirement for the overall system. When vector, x, of positive integers conforms to the nonlinear constraint $R(x) \ge R$, the corresponding system meets or exceeds the requirement for reliability. An optimal solution shows how to achieve the required reliability with the fewest identical components.

The main result of Section 2 characterizes optimal solutions of Model 1 in terms of the quotient and remainder when z, the number of available components, is divided by n, the number of subsystems. The optimal objective value depends on all three parameters, n, z, and ρ, while the optimal vectors depend only on the parameters n and z. The model has unique optimal vector if and only if z is a multiple of n.

The main result of Section 3 describes the optimal solutions of Model 2 for which the overall reliability is as high as possible. Chern [1] noted that this special case of redundancy allocation models can be solved with a polynomially bounded algorithm. Section 4 includes an elementary example of such an algorithm. Section 5 includes examples and concluding observations.

2 Allocating z Identical Components among n Independent Subsystems in Series to Maximize System Reliability

If x is an optimal vector for Model 1, then $\sum_{i=1}^{n} x_i = z$ because an allocation that leaves components unused can be improved by adding a component to any of the subsystems. Among allocations of exactly z identical components among n subsystems, those in which the largest subsystem has at least two more components than the smallest cannot be optimal. In fact, as shown in the next lemma, simply switching one component from the largest subsystem to the smallest will improve the reliability without changing the total number of components.

Lemma 2.1. *Let $x = (x_1, x_2, \cdots, x_n)$ be an $n-$vector of positive integers for which $\sum_{i=1}^{n} x_i = z$. If $n \geq 2$ and the difference between the largest and smallest coordinates of x is at least 2, then there is a vector, y, of positive integers with $\sum_{i=1}^{n} y_i = z$. and $R(x) < R(y)$.*

Proof. Choose indices i' and j' for which $x_{j'} - x_{i'} \geq 2$. Let y be the vector that agrees with x except at these two indices, where $y_{i'} = x_{i'} + 1$ and $y_{j'} = x_{j'} - 1$. The coordinates of y are also positive integers and $\sum_{i=1}^{n} y_i = z$. Since $x_{j'} - 1 > x_{i'}$, it follows that $(1-\rho^{x_{i'}})(1-\rho^{x_{j'}}) < (1-\rho^{y_{i'}})(1-\rho^{y_{j'}})$ and so, $R(x) < R(y)$. ☐

Thus, at an optimal solution of Model 1, the difference between the largest and smallest coordinates is as small as possible. In the context of redundancy allocation, the lemma formalizes the idea that among all allocations of z identical components to n subsystems, the highest reliability is achieved when the sizes of the subsystems are as closely balanced as possible Baxter and Harche [5].

With the division algorithm, any positive integer, z, can be written as $qn + r$, where q is a nonnegative integer and r is an integer in the set $\{0, 1, \cdots, n-1\}$. For the parameters of Model 1, the quotient, q, is strictly positive because $z \geq n$. The next theorem shows that the integers q and r determine the maximum reliability of a system formed by allocating z identical components among n subsystems.

Theorem 2.2. *If $x = (x_1, x_2, \cdots, x_n)$, is a vector of positive integers, with $\sum_{i=1}^{n} x_i = qn + r$, where q is a positive integer and r is an integer in the set $\{0, 1, \cdots, n-1\}$, then*

$$R(x) \leq (1-\rho^q)^{n-r}\left(1-\rho^{q+1}\right)^r.$$

Proof. For integers q and r as in the hypotheses, the set of positive integral vectors, x, on the hyperplane defined by $\sum_{i=1}^{n} x_i = qn + r$, is finite and non-empty. Let x' denote a positive vector at which the function $R(x)$ achieves its greatest value on this set. From the previous lemma, it follows that the difference between the largest and smallest coordinates of x' is at most 1.

If $r = 0$, then the smallest coordinate of x' is no greater than q and the largest is no smaller than q. However, since the sum of the coordinates is equal to qn, if one of the largest or the smallest coordinate of x' is different from q, then so is the other. But, then the difference between the largest and smallest coordinates is at least 2. So, if $r = 0$, then $x' = (q, q, q, \cdots, q)$.

If $r > 0$, the largest coordinate of x' is at least $q + 1$ and the smallest is at most q. Because the difference between the largest and smallest coordinates is at most 1, the smallest coordinate is equal to q and the largest is equal to $q + 1$; in addition, exactly r coordinates are equal to $q + 1$. There are $\binom{n}{r}$ vectors at which the best reliability is achieved. $\square$

3 Determining the Minimal Number of Identical Components to Achieve a Reliability Goal

Although the feasible set for Model 2 includes infinitely many vectors with positive, integral coordinates, the search for an optimal solution can be limited to a finite set.

Lemma 3.1. *Model 2 has an optimal solution and the optimal objective value is no greater than* $n\left\lceil \dfrac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil$.

Proof. The positive, integral vector in which each coordinate is equal to $\left\lceil \dfrac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil$ satisfies the reliability constraint of Model 2, and so the minimal number of components which are required to achieve the reliability goal is at most $n\left\lceil \dfrac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil$. Since the set of positive, integral vectors for which

$R(x) \geq R$ and $\displaystyle\sum_{i=1}^{n} x_i \leq n\left\lceil \dfrac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil$ is finite and non-empty, there is at least one vector at which the sum achieves its minimum. $\square$

Replacing the integrality requirements by the constraints $x_i > 0$ for $1 \leq i \leq n$, Moskowitz and McLean [6] solved the continuous relaxation of Model 2 by the method of Lagrange multipliers and then obtained the positive integral vector of Lemma 3.1 by rounding up to the next integer, each coordinate of the optimal solution of the relaxed model.

Definition 3.2. If $(1-\rho)^n < R$, then the integer $\left\lceil \dfrac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil$ is greater than 1, and the positive integer $q*$ is defined by the equation

$$\left\lceil \frac{\log_2(1-R^{\frac{1}{n}})}{\log_2(\rho)} \right\rceil = 1 + q^*.$$

For integers r, with $0 \leq r \leq n$, the products $R_r = \left(1-\rho^{q^*+1}\right)^r \left(1-\rho^{q^*}\right)^{n-r}$ are strictly increasing, with $R_n \geq R > R_0$. The positive integer r^* is defined by

$r^* = \min\{r : 0 < r \leq n \text{ and } R_r \geq R\}$.

Theorem 3.3.

(i) *If* $(1-\rho)^n \geq R$, *then Model 2 has the unique optimal solution* $x^* = (1, 1, \cdots, , 1)$ *and the optimal objective value is equal to* n.

(ii) *If* $(1-\rho)^n < R$ *and* $r^* = n$, *then the optimal objective value for Model 2 is equal to* $n(1+q^*)$ *and the vector for which each coordinate is equal to* $1+q^*$ *is an optimal vector.*

(iii) *If* $(1-\rho)^n < R$ *and* $r^* < n$, *then the optimal objective value for Model 2 is equal to* $nq^* + r^*$ *and the vector for which each of the first* r^* *coordinates is equal to* $1+q^*$ *and each of the last* $n-r^*$ *coordinates is equal to* q^* *is an optimal vector.*

Proof. If $(1-\rho)^n \geq R$, then the reliability goal can be achieved without redundancy.

If $(1-\rho)^n < R$, it follows from Theorem 2.2 that, among the redundancy allocations represented by positive integral vectors whose sum is no greater than nq^*, the best reliability is equal to $(1-\rho^{q^*})^n$, which is too small to conform to the reliability constraint of Model 2. From this observation and Lemma 3.1, the optimal objective value, z^*, for Model 2 can be written as $nq^* + r$, with $0 < r \leq n$. From Theorem 2.2, it follows that R_r is the best reliability that can be achieved with $nq^* + r$ components and that the configuration represented by the vector in the statement of the theorem achieves this reliability. Among these configurations, the smallest number of components in a feasible configuration is equal to $n. + r^*$. □

Barlow and Proschan [10] produced highest-reliability solutions of Model 2 from the initial vector $(1, 1, \cdots, 1)$ by adding components one at a time until the reliability goal is achieved. Their criterion for choosing the subsystem to which the next component is added is a ratio test, which, when the cost coefficients are all equal to 1 and the failure probabilities of the components are constant across the subsystems, never permits the numbers of components in any pair of subsystems to differ by more than one. The next section includes a faster method for building this optimal configuration.

4 Computing Optimal Solutions with Rational Arithmetic

For Model 1, which shows how to use redundancy to maximize reliability, the optimal allocation of z components among n subsystems is determined by the quotient and remainder when z is divided by n. If z is a multiple of n, then the optimal configuration is unique each subsystem has $\frac{z}{n}$ components. If $z = qn + r$, with $1 \leq r \leq n-1$, each optimal configuration has r subsystems with $q+1$ components and $n-r$ subsystems with q components.

For Model 2, Theorem 3.3 shows the optimal configurations for which the difference between the largest and the smallest coordinates is as small as possible. For an integer, n, with $n \geq 2$ and rational values of R and ρ, the integers q^*+1 and r^* of Theorem 3.3 can be computed relatively quickly with elementary arithmetic operations. Proposition 4.1 shows how to find q^*+1 by starting with $m = 0$ and computing reliability of systems with 2^m components in each subsystem until the reliability goal is achieved and then backtracking to find the smallest system that meets the reliability goal with subsystems all having the same size.

Proposition 4.1. *For an integer,* n, *greater than 1, and rational numbers,* ρ *and* R, *with* $0 < \rho < 1$, $0 < R < 1$ *and* $(1-\rho)^n < R$, *the integer* q^*+1 *can be computed with rational arithmetic in at most* $2\lceil \log_2(1+q^*) \rceil$ *steps.*

Proof. For fixed values of the parameters n and ρ, the function defined on the positive integers by $x \longmapsto (1-\rho^x)^n$ takes values in $(0,1)$ and is strictly increasing with respect to x. As

$x \longmapsto \infty$, $(1-\rho^x)^n$ increases to 1, so the set of positive integers for which $(1-\rho^x)^n \geq R$ is not empty. From Definition 3.2, it can be seen that q^*+1 is the least integer in this set.

Set $t_0 = \rho$ and $m_0 = 1$. As long as $(1-t_k)^n < R$, compute $t_{k+1} = t_k^2$ and

$m_{k+1} = 2m_k$. Since the terms t_k decrease to 0, the first index for which $(1-t_k)^n \geq R$ is a well-defined, positive integer, which we denote by K. Notice that $(1-t_k)^n$ is equal to the reliability of the series system in which each subsystem consists of 2^k components configured with active redundancy. As constructed, the integer K is the smallest exponent for which such a series system conforms to the reliability constraint. Thus, $2^{K-1} < 1+q^* \leq 2^{K}$, so $K = \lceil \log_2(1+q^*) \rceil$.

If $K = 1$, then $1+q^* = 2$. If $K > 1$, set $U_0 = m_K$, $L_0 = m_K/2$, and observe that $L_0 < 1+q^* \leq U_0$. Next, set $\delta_0 = m_K/4$, $M_0 = L_0 + \delta_0$ and $u_0 = t_{K-1}t_{K-2}$. Notice that u_0 is equal to the reliability of the series system in which each subsystem consists of M_0 components configured with active redundancy. If $u_0 < R$, then $M_0 < 1+q^* \leq U_0$; if $u_0 \geq R$, then $L_0 < 1+q^* \leq M_0$. In particular, if $K = 2$, then $1+q^* = 3$ if $(1-u_0)^n \geq R$, and $1+q^* = 4$ if $(1-u_0)^n < R$.

For larger values of K, the integer $1+q^*$ can be found by repeating this construction. If $K > 2$, then, as long as $1 \leq j \leq K-2$, if $(1-u_{j-1})^n < R$, set $U_j = U_{j-1}$, $L_j = M_{j-1}$ and $u_j = u_{j-1}t_{K-2-j}$; if $(1-u_{j-1})^n \geq R$, set $U_j = M_{j-1}$, $L_j = L_{j-1}$ and $u_j = u_{j-1}/t_{K-2-j}$. For $1 \leq j \leq K-2$, set $\delta_j = \delta_{j-1}/2$, $M_j = L_j + \delta_j$. At stage j of the procedure, $L_j < 1+q^* \leq U_j$, $U_j - L_j \leq 2^{K-1-j}$ and u_j is equal to the reliability of the series system in which each subsystem consists of M_j components configured with active redundancy. At stage $K-2$, the integers L, M and U are consecutive. If $(1-u_{K-2})^n < R$, then $1+q^* = U_{K-2}$; otherwise, $1+q^* = M_{K-2}$. □

Once the value of q^* is fixed, the products R_r of Definition 3.2 are strictly decreasing as r runs from n down to 0. The set of positive integers for which $R_r \geq R$ is non-empty and its smallest element is r^*, which can be computed by the bisection algorithm given in Proposition 4.2.

Proposition 4.2. *For an integer n, with $n \geq 2$, and rational numbers, ρ and R, with $0 < \rho < 1$, $0 < R < 1$ and $(1-\rho)^n < R$, the integer r^* can be computed with rational arithmetic in at most $2\lceil \log_2(n) \rceil$ steps.*

Proof. Set $t'_0 = (1-\rho^{q^*})/(1-\rho^{q^*+1})$ and $m'_0 = 1$. As long as $t'_K(1-\rho^{q^*+1})^n \geq R$, set $t'_{k+1} = (t'_k)^2$ and $m'_{k+1} = 2m'_k$. Notice that $t'_K(1-\rho^{q^*+1})^n = R_r$ for $r = n - 2^K$. Since $t'_0 < 1$, the terms $t'_K(1-\rho^{q^*+1})^n$ decrease to 0, so the first index for which $t'_K(1-\rho^{q^*+1})^n < R$ is a well-defined, non-negative integer, denoted by K'. Since $(1-\rho^{q^*})^n < R$ and $m'_K = 2^K$, $K' \leq \lceil \log_2(n) \rceil$. Notice that $t'_K(1-\rho^{q^*+1})^n = R_r$ for $r = n - 2^K$.

If $K' = 0$, then $r^* = n$; if $K' = 1$, then $r^* = n-1$. Otherwise, set $U_0 = m'_K$ $L_0 = m'_K/2$, $\delta_0 = m'_K/4$, $M_0 = L_0 + \delta_0$ and $u'_0 = t_{K'-1}t_{K'-2}$. If $K' = 2$, then, $r^* = n-3$ if $u'_0(1-\rho^{q^*+1})^n \geq R$, and $r^* = n-2$ if $u'_0(1-\rho^{q^*+1})^n < R$. If $K' > 2$, then, as long as $1 \leq j \leq K'-2$, if $u'_{j-1}(1-\rho^{q^*+1})^n < R$, set $U_j = M_{j-1}$ $L_j = L_{j-1}$ and $u'_j = u'_{j-1}/t_{K'-j-2}$; if $u'_{j-1}(1-\rho^{q^*+1})^n \geq R$, set $U_j = U_{j-1}$, $L_j = M_{j-1}$ and $u'_j = u'_{j-1}t'_{K-2-j}$. As long as $1 \leq j \leq K'-2$, $\delta_j = \delta_{j-1}/2$, and $M_j = L_j + \delta_j$. If $u_{K'-2}(1-\rho^{q^*+1})^n < R$, then $n - r^* = L_{K'-2}$; otherwise, $n - r^* = M_{K'-2}$. □

5 Examples and Conclusions

For Model 1 and a set of parameters n, ρ and z, the quotient q and remainder r obtained by applying the division algorithm to the dividend z and the divisor n determine all optimal solutions. For $z = nq + r$ with $0 \le r < n$, the optimal objective value is equal to $\left(1 - \rho^{q+1}\right)^{r} (1 - \rho^{q})^{n-r}$. If $r = 0$, then the model has only one optimal solution the vector in which each component is equal to the quotient q. If $r > 0$, then the model has a unique optimal vector in which the coordinates are non-increasing: namely, the vector in which the first r coordinates are equal to $q + 1$ and the last $n - r$ are equal to q. This configuration can be rearranged to produce $\binom{n}{r}$ optimal solutions.

Likewise for Model 2, when $(1 - \rho)^n < R$, the coordinates of the vector x^* of Theorem 3.3 may be rearranged to produce $\binom{n^*}{r^*}$ optimal vectors, each with the highest reliability that can be achieved with z^* components. Table 5.1 summarizes some optimal solutions achieved by the methods of Section 4. These show how the optimal objective value and the number of optimal solutions with the highest reliability respond to changes in the parameters ρ and n.

For the special case in this paper, the solutions with $q^* + 1$ components and q^* components in each subsystem are exactly the integral solutions obtained by rounding up and down the solution of the Lagrangian relaxation. As a result, even in this special case, the bound $(n - 1)$ is the best that can be guaranteed for the number of hyperplanes to be searched if one proceeds as in Nmah[7]. The big improvement for the special case in this paper is that only one integral solution is tested on each hyperplane, as opposed to the much larger bound in Nmah[7] for the general case.

In Nmah [8] and Nmah [9], Nmah presents some explicit examples of multiple optimal solutions with different values for system reliability. While the algorithm given in Nmah [7] will find all optimal solutions in situations where the problems of discourse have multiple optimal solutions, the algorithm in this paper finds quickly only those with the highest reliability as demonstrated in Table 5.1 and Table 5.2. Table 5.2 contains solutions for the case where the parameter R as well as the other parameters varies in values.

Table 5.1. Highest-reliability integral solutions

n	ρ	$q^* + 1$	r^*	$z^* = nq^* + r^*$
2	0.1	3	2	6
2^2	0.1	3	4	12
2^4	0.1	4	7	55
2^8	0.1	5	173	1197
2^{16}	0.1	7	61651	454867
2	0.5	8	2	16
2^2	0.5	9	3	35
2^4	0.5	11	12	172
2^8	0.5	15	183	3767
2^{16}	0.5	23	46764	1488556

$$\text{Min} \sum_{i=1}^{n} x_i, \text{ Subject to } \prod_{i=1}^{n} (1 - \rho^{x_i}) \ge 0.99, \ x_i \text{ integral}, \ x_i \ge 1$$

Barlow and Proschan [10] had observed that their method of adding one component at a time would produce the highest-reliability solutions of Model 2, but could pass over other optimal solutions. For example, any n-vector, x, with $\sum_{i=1}^{n} x_i = nq^* + r^*$ and $R \leq R(x) < R_{r^*}$, will be an optimal solution of Model 2. Nmah [11] has developed an algorithm for finding all alternate optimal allocations of identical components of Model 2.

Table 5.2. Highest-reliability integral solutions

n	ρ	R	$q^* + 1$	r^*	$z^* = nq^* + r^*$
2	0.1	0.99	3	2	6
2^2	0.1	0.99	3	4	12
2^4	0.1	0.99	4	7	55
2^8	0.1	0.99	5	173	1, 197
2^{16}	0.1	0.99	7	61, 651	454, 867
2	0.1	0.999	4	2	8
2^2	0.1	0.999	4	4	16
2^4	0.1	0.999	5	7	71
2^8	0.1	0.999	6	174	1, 454
2^{16}	0.1	0.999	8	61, 702	520, 454

$$\text{Min} \sum_{i=1}^{n} x_i, \text{ Subject to } \prod_{i=1}^{n} (1 - \rho^{x_i}) \geq R, \; x_i \text{ integral}, \; x_i \geq 1$$

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Competing Interests

Author has declared that no competing interests exist.

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