



# Variations of the Time to Failure (TTF) for Specific Components in Aeronautical Navigation Systems

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## Authors' contributions

This work was carried out in collaboration between the two authors. Author AMA performed the analysis, solved the example, wrote the first draft of the manuscript and initiated the literature search. Author AMR envisioned and designed the study, contributed to the symbolic and numerical analysis, checked the solution of the example, managed and finalized the literature search, and substantially edited and improved the entire manuscript. Both authors read and approved the final manuscript.

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## ABSTRACT

Aeronautical navigation systems installed at specific locations within Saudi Arabia report the actual observed data of the times to failure (TTFs) for many types of engineering elements made by some typical international highly-acclaimed manufacturers. There is a need to compare these observed data to the estimated data provided by the manufacturers themselves. The paper gathers a lot of data on the actual failure instances of many components produced by a variety of manufacturers and installed at different locations within the extensive area of Saudi Arabia. These data are used to calculate the mean times between failures for these components. The paper points to an inadvertent discrepancy between these data and the corresponding mean times between failures (MTBFs) suggested by some prominent manufacturers. Such suggested MTBFs are typically optimistic and unrealistically high irrespective of the elements, the location, and the manufactures. The work reported herein is a preliminary assessment of this phenomenon that might lead to its theoretical modeling and subsequent understanding.

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## 1. INTRODUCTION

This paper reports a phenomenon concerning many types of components used in aeronautical navigation systems in Saudi Arabia. This phenomenon is unusual, strange, peculiar, surprising, and difficult to understand or explain. For certain manufacturers, there is a *significant* disagreement between the observed times between failure (TBFs) of many types of components and the corresponding MTBFs suggested by the manufacturers, regardless of the location in which the component is being used. Such a disagreement is always in the *undesirable* direction, i.e., the observed TTFs are always lower (and sometimes significantly much lower) than the manufacturer's suggested MTBF. In fact, the observed data do not look like data obtained under normal operating conditions, but are similar to data obtained under burn-in testing or accelerated failure [1-5]. Ironically enough, this observed phenomenon is in direct contradiction with (a) a common practice in many developing countries, wherein attempts are made to maintain and support complex systems that are being operated beyond their designed life [6,7], and (b) the situation where obsolescence is a major concern, such as when dealing with legacy electronic military systems [8,9].

This paper attempts to establish the existence of the aforementioned phenomenon, a preliminary step towards an extensive exploration, hopefully leading to full understanding of it, which might include or lead to:

- The assertion of a plausible explanation of the phenomenon, and whether it is due to harsh working conditions, severe desert climate, ineffective maintenance policies, optimistic (or even fraudulent) prediction by some of the manufacturers, or something else.
- The construction of necessary mathematical or statistical models to capture the essence of this phenomenon.
- The comparison of the model predictions and the actual data for various components and locations.
- The comparison of the situation in the Aeronautics Sector with the situation in other Industrial Sectors.

- The comparison of the situation in Saudi Arabia with that in other countries of similar climates, and other countries worldwide.
- The forwarding of a model-based mathematized theory that gives an acceptable interpretation of the real-life data.
- The assessment of inherent economic losses associated with the aforementioned phenomenon.
- The decision whether the concerned manufacturer is to be blamed for this discrepancy, i.e. whether the manufacturer officers made their predictions on good faith using agreed-upon scientific methodology in a competent and honest manner, and, if not, whether they were scientifically incompetent or they deliberately cheated so as to sell an inferior product for an undeserved high price.

The main theme or premise of this paper might be compatible with ongoing discussions that attempt to correlate degradation of reliability metrics (such as the MTBF) to the existence of adverse or even hostile operating conditions [10,11]. Turl and Wood [12] observe that "all too often, laboratory-proven, sophisticated and effective instrumentation can suffer from degraded performance, and even failure, when deployed in demanding or harsh conditions." If this premise does not suffice to justify the observed phenomenon, then doubts about the competency or integrity of manufacturers cannot be avoided, though these doubts cannot be raised to the status of public or formal accusations. In fact, the possibility of incompetency or even deliberate fraud cannot be entirely ruled out [13-15]. However, since the burden of proof will be on the customer and not on the manufacturer, the achievement of due justice will face a long way to go. Cressey [16] implicitly asserts the existence of management fraud in our contemporary world, and hopes it will decline in frequency and severity when we develop knowledge about its causes and then use that knowledge in prevention programs.

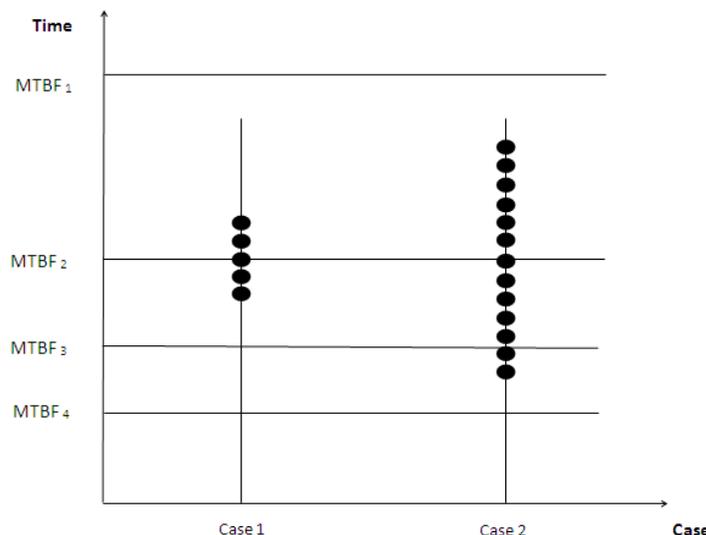
The organization of the rest of this paper is as follows. Section 2 presents an overall picture of the possibilities of the relation between actual observed data and predicted data. Section 3

reports many samples of actual observed data, individually for singled-out components used in specific locations, and then collectively by aggregating the data for many components, several locations and a variety of manufacturers. Section 4 concludes the paper. The paper relies heavily on some technical reliability terminology that might be found in many texts and papers (see, for example, [17-28]). Appendix A is a handy glossary for some of the technical nomenclature used in the paper, and explains some of the underlying terms and mathematical relations to make the paper self-contained. The reason of relegating this material to the appendix (rather than incorporating it within the main text) is to avoid distracting the reader from the essence of the message conveyed by the paper. We do not want our readers to fail to see the forest for the trees, or to get too involved in the details to observe the picture as a whole.

## 2. POSSIBILITIES OF THE RELATION BETWEEN ACTUAL AND PREDICTED DATA

The actual data recorded for each component at a specific location provides the instant  $T_i$  at which the  $i$ th failure occurs. We need a subtraction operation to obtain the times between two successive failures  $TBF_i$ , namely

$$TBF_i = T_i - T_{i-1}, \quad i \geq 1, T_0 = 0. \quad (1)$$



**Fig. 1. Hypothetical scatter of actual TTF data in two cases: Case 1 (narrow spread, good precision), and Case 2 (wide spread, low precision). The data is compared to four estimates of the MTBF submitted by the manufacturer: (a)  $MTBF_1$  (optimistic prediction), (b)  $MTBF_2$  (accurate prediction), (c)  $MTBF_3$  (conservative prediction), and (d)  $MTBF_4$  (overly conservative or too pessimistic prediction)**

Implicit in the above equation are typical assumptions of reliability theory [29,30], including the following assumptions

- (a) The component is perfect when it is new, i.e., the reliability  $R(t)$  satisfies.

$$R(t) \Big|_{t=0} = 1.0 \quad (2)$$

- (b) When the component fails, its failure is detected and reversed (via replacement or repair) in a supposedly *negligible* time, such that the component becomes as good as new, and hence the reliability jumps again to 1.0 when a failure (with subsequent repair or replacement) occurs, i.e.,

$$R(t) \Big|_{t=T_i} = 1.0 \quad (3)$$

Fig. 1 displays a hypothetical scatter of TTF data (of a specific component at a given location) in two cases: Case 1 depicts a narrow spread of data (good precision), while Case 2 has a wide spread of data (low precision). The data is compared to four estimates of the MTBF that are possibly submitted by the manufacturer:

- (a)  $MTBF_1$  (optimistic prediction),
- (b)  $MTBF_2$  (accurate prediction),
- (c)  $MTBF_3$  (conservative prediction), and
- (d)  $MTBF_4$  (overly conservative or too pessimistic prediction).

In the next section, we report actual data, taken from the Database of System Failure, Maintenance Control Administration, Aeronautical Navigation Services, Saudi Arabia [31]. We try to see how this data compares with that in Fig. 1, and subsequently classify it as one of the four types suggested in the figure. We note that, strangely enough, optimistic prediction is prevalent for many components, many locations, and many manufacturers. We have deliberately used the neutral non-aggressive non-offensive label of ‘optimistic’ for a kind of prediction that uses an exaggerated value for the MTBF. We refrain from giving any judgment on such a prediction, which might or might not be scientifically and morally sound. Our purpose is simply to alert researchers to our large-scale observation. Definitely, further investigations are needed to replicate our results, and to make them more

specific and quantitative. Whether these results have ramifications concerning competency or honesty is still an open question. Further extensive study, based on advanced statistical methods, is warranted for answering this question.

### 3. SAMPLES of ACTUAL OBSERVED DATA

We start by the good or normal news first. Figs. 2(a)–2(c) show the TTF variation with locations for specific components made by certain manufacturers, with the MTBF suggested by the manufacturer appearing as a constant or horizontal line independent of the location, and fortunately, it is almost a lower bound for the observed data. The manufacturer’s suggestions here are rather *conservative* and this is what is expected of a manufacturer who is

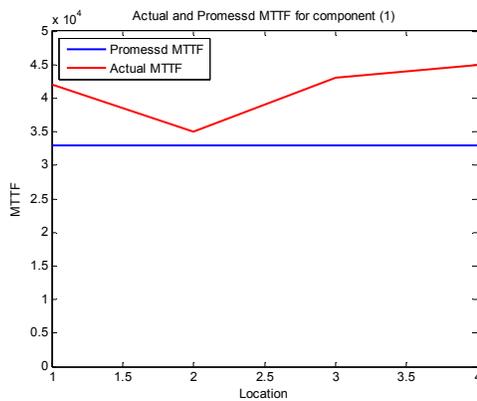


Fig. 2.(a)

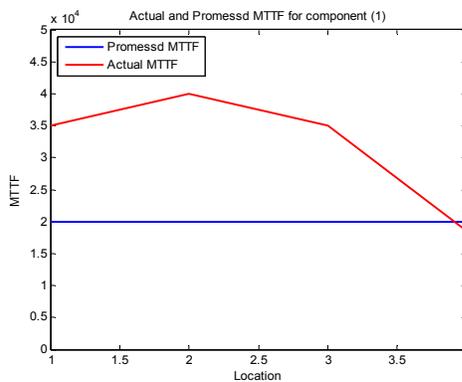


Fig. 2.(b)

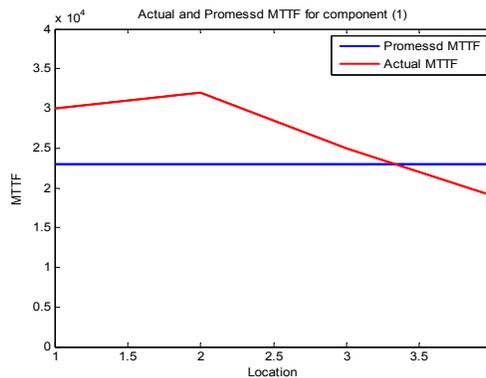


Fig. 2.(c)

Fig. 2 (a) – (c). MTTF variation with locations for specific components made by a certain manufacturer. The MTTF estimated and promised by the manufacturer (shown in blue) is constant irrespective of the location, while the actual MTTF observed by the operators (depicted in red) varies with location

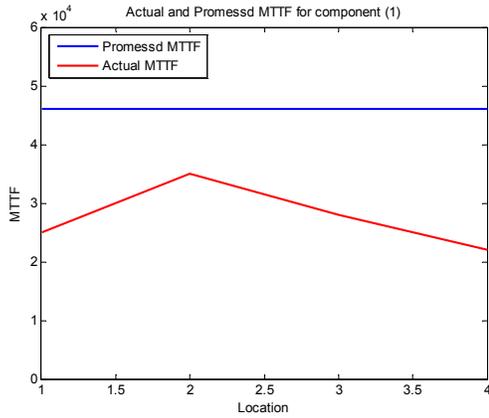


Fig. 3.(a)

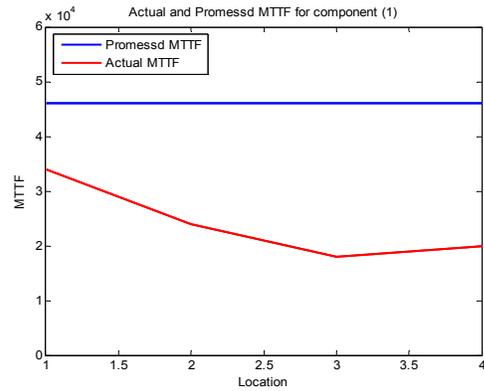


Fig. 3.(b)

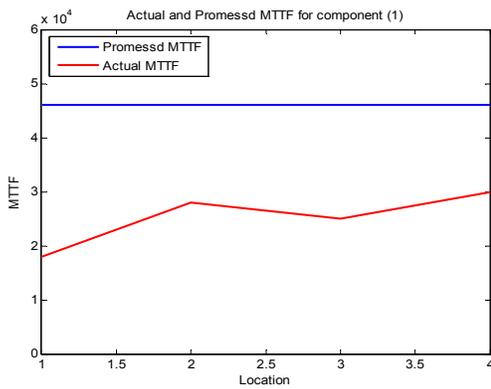


Fig. 3.(c)

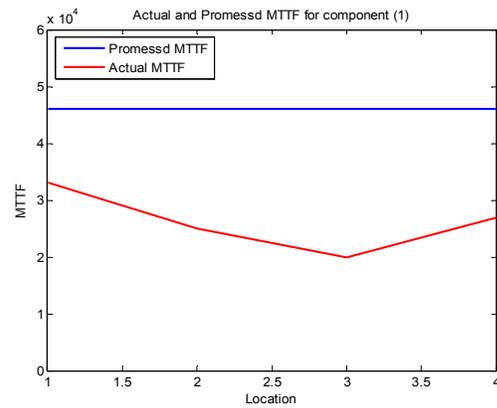


Fig. 3.(d)

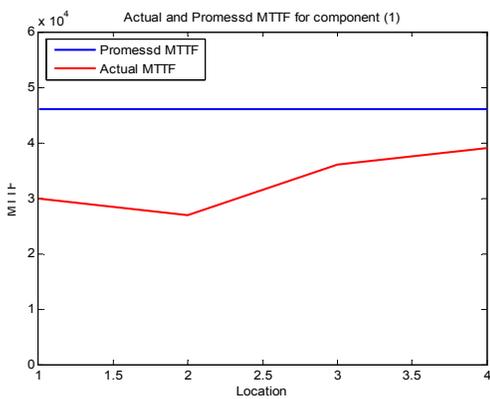


Fig. 3.(e)

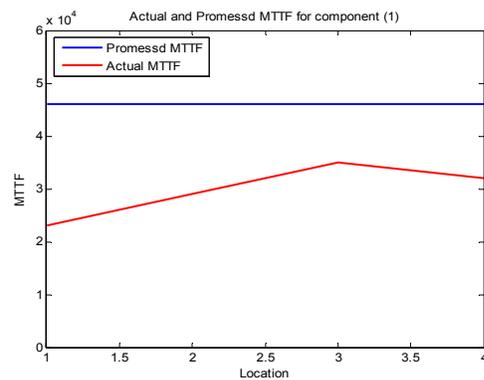


Fig. 3.(f)

Fig. 3 (a) – (f). Case when the MTBF suggested by the manufacturer sets an upper bound for the graph of MTTF variation with location

- a. scientifically and technically competent.
- b. of sound moral integrity.
- c. knowledgeable about its target market, and taking into consideration the harsh operating conditions and the severe desert climate in most regions of the Arabian Peninsula.

Figs. 3(a)–3(f) demonstrate the phenomenon we need to report. Here, the constant MTBF suggested by the manufacturer sets an upper bound for the graph of TTF variation with location. It is alarming to see that the situations depicted in Figs. 3(a)–3(f) are much more frequent than those noted in Figs. 2(a)–2(c).

#### 4. CONCLUSIONS

This paper presents a thought-provoking observation on the validity of manufacturer's own assessment of their own products. While a few particular manufacturers give conservative predictions of the MTBFs for their components, some other manufacturers give optimistic and occasionally, too optimistic predictions. As we pointed out in the introduction, there are potentially many potential subsequent studies and several ramifications for the phenomenon observed herein.

In passing, we note that the manufacturer's supplied data is restricted to the MTBF which is the same as the MTTF under the assumption that a component becomes as good as new when repaired, and that repair time is negligible. This is a questionable practice, since the random variable TTF cannot be adequately described by a single measure of its central tendency, namely, the mean. In engineering, one needs, at least another metric that measures the spread about this mean, namely the variance, or its square root (the standard deviation). Statisticians might be more demanding and ask for the next central moments represented by the coefficient of skewness and the coefficient of kurtosis [32-47].

A misconception concerning the MTBF is widespread, which is due to its being only a rough estimate of the life of an individual part. Using the MTBF to predict the time to failure (TTF) of a single part is fundamentally flawed, because the MTBF does not apply to a specific part [48]. Using the MTBF as a synthetic indicator of reliability often represents the wrong choice, albeit its use is often imposed by regulations or contracts [49]. Therefore, alternative structured ways of communicating

about reliability, based on further concepts than the bare MTBF, are currently being proposed [49-51].

#### COMPETING INTERESTS

The authors have declared that no competing interests exist.

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## APPENDIX A

The bath-tub curve representing the failure rate (hazard rate) versus time consists of three operation intervals namely; (a) the debugging (burn-in, or infant mortality) interval, (b) the prime-of-life (useful life) interval, and (c) the wear-out interval. These three intervals in the bath-tub curve correspond to a decreasing failure rate (DFR), a constant failure rate (CFR), and an increasing failure rate (IFR), respectively [17-28].

Reliability  $R(t)$  of a system (component) is defined as the probability that the system (component) will function over the interval  $(0, t]$  (provided it was functioning at  $t = 0$ ). It is related to the random variable  $T$  representing time to failure ( $T \geq 0$ ) by [23, 25, 43, 44, 52-58]:

$$R(t) = \Pr \{T \geq t\} = 1 - F(t) \quad (4)$$

Where  $F(t)$  is the Cumulative Distribution Function (CDF) of the random variable  $T$ . Therefore, we have:

$$f(t) = dF(t)/dt \quad (5)$$

as the probability density function (pdf) of  $t$ . The MTTF (life expectancy) for a non-repairable system (component) is given by:

$$MTTF = E\{T\} = \int_0^{\infty} R(t) dt \quad (6)$$

If we adopt the somewhat controversial assumption [59-64] that a non-repairable component has a constant failure rate (CFR)  $\lambda$ , then the reliability of that component follows an exponential distribution as  $p(t) = e^{-\lambda t}$ , and hence its life expectancy is simply the reciprocal of this CFR, namely:

$$[MTTF]_{\text{component}} = \int_0^{\infty} [p(t) dt] = \int_0^{\infty} [e^{-\lambda t} dt] = \lambda^{-1}, \quad (7)$$

The MTTF has a “dual” quantity called the mean-time-to-repair (MTTR), which is usually presumed negligible for critical systems, for which repair might amount to immediate replacement. Together the two quantities determine the steady-state availability of a repairable system as  $MTTF / (MTTF + MTTR)$ , and its steady-state unavailability as  $MTTR / (MTTF + MTTR)$ . While reliability denotes success within the interval  $(0, t]$ , availability denotes instantaneous success at moment  $t$  [29].

If the system (component) is repairable, it alternates between good (up) and failed (down) states, as it experiences an alternating sequence of failures (switching it from up to down) and repairs (recovering it from down to up) The concepts of time-to-failure (TTF) and its mean (MTTF) are then generalized to those of time-between-failures (TBF) and its mean (MTBF). The mean-time-between-failures (MTBF) is the average time between system breakdowns, and is a crucial maintenance metric, especially for critical or complex systems, such as those involved in the aeronautical industry. For these systems, the metric MTTF is still used, but now it might denote the mean time to first failure.

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