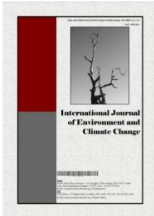

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Estimation by Inverse Approach of a Transmissivity Field over the Entire Continental Terminal Aquifer of Abidjan

Kouamé Auguste Kouassi[1*], Francis Williams Kouassi[1], Oi Mangoua Jules Mangoua[2], Philippe Ackerer[3], Gountôh Aristide Douagui[1] and Issiaka Savane[1]

[1]UFR Environment, Laboratory of Environmental Sciences and Technologies, Jean Lorougnon Guedé Daloa University, BP 150 Daloa, Côte d'Ivoire.
[2]UFR Sciences and Environmental Management, Geosciences and Environment Laboratory, Nangui Abrogoua University, 02 BP 801 Abidjan 02, Côte d'Ivoire.
[3]Department 'Laboratoire d'Hydrologie et de Géochimie de Strasbourg', Strasbourg University, France.

Authors' contributions

This work was carried out in collaboration among all authors. Author KAK designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors FWK, OMJM and PA managed the analyses of the study. Authors GAD and IS managed the literature searches. All authors read and approved the final manuscript.

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ABSTRACT

Hydraulic characterization of aquifer systems is important for the development of exploitation scenarios and groundwater management strategies. Especially in lithologically heterogeneous aquifers, local scale variations in transmissivity (T) may not be neglected. Field scale transmissivity values are usually derived from pumping tests, but in most cases their number and availability are rather limited. Thus, direct measurement of transmissivity over an entire aquifer is expensive and technically almost impossible. In such situations, inverse hydrodynamic modelling is the

**Corresponding author: E-mail: kouameauguste@yahoo.fr, kouassiauguste.sge@univ-na.ci;*

appropriate solution. In this article, the real transmissivity field of the aquifer of the Continental Terminal of Abidjan is investigated by a multi-scale parametrization that allows to bypass the problem of scale change and to determine this hydrodynamic parameter over the entire aquifer. This hydrogeological modelling of the Continental Terminal aquifer identified a structure of 153 nodes in size as the closest structure to that of the Continental Terminal aquifer. The transmissivity field associated with this optimal size, ranging from $5.4.10^{-5}$ to 1 m^2s^{-1}, has been compared with values published in other studies in Africa and the world. These identified values are plausible and have a good overall structure. The success of this modeling is strongly linked to the quantity, quality and spatial distribution of authentic informations on the parameters sought.

Keywords: Aquifer of Abidjan Continental Terminal; dimension; multiscale; parametrization; structure; transmissivity.

1. INTRODUCTION

The numerous demands on water, in a context of population explosion, industrialization and intensive agriculture, have the consequence of reducing the water supply available everywhere in the world [1]. Abidjan, the fourth largest city in Africa and the largest city in Ivory Coast, is no exception to this global reality. It represents less than 1% of the national territory and hosts one fifth of the Ivorian population with an annual growth rate of 2.6% (RGPH, 2014), but produces and consumes almost 70% of the country's drinking water [2]. Abidjan is mainly supplied with potable water from the groundwater of the Continental Terminal, also known as the "Abidjan groundwater". This aquifer, covering an area of about 1215 km², is limited to the North by the contact between crystalline basement and sedimentary basin in the Anyama agglomeration, to the South by the edge of the Ebrié lagoon, to the East by the Mé and the Potou lagoon and to the West by the Agnéby river and its affluent, Niéké (Fig. 1). The Continental Terminal also includes four sedimentary horizons generally unconfied [3].

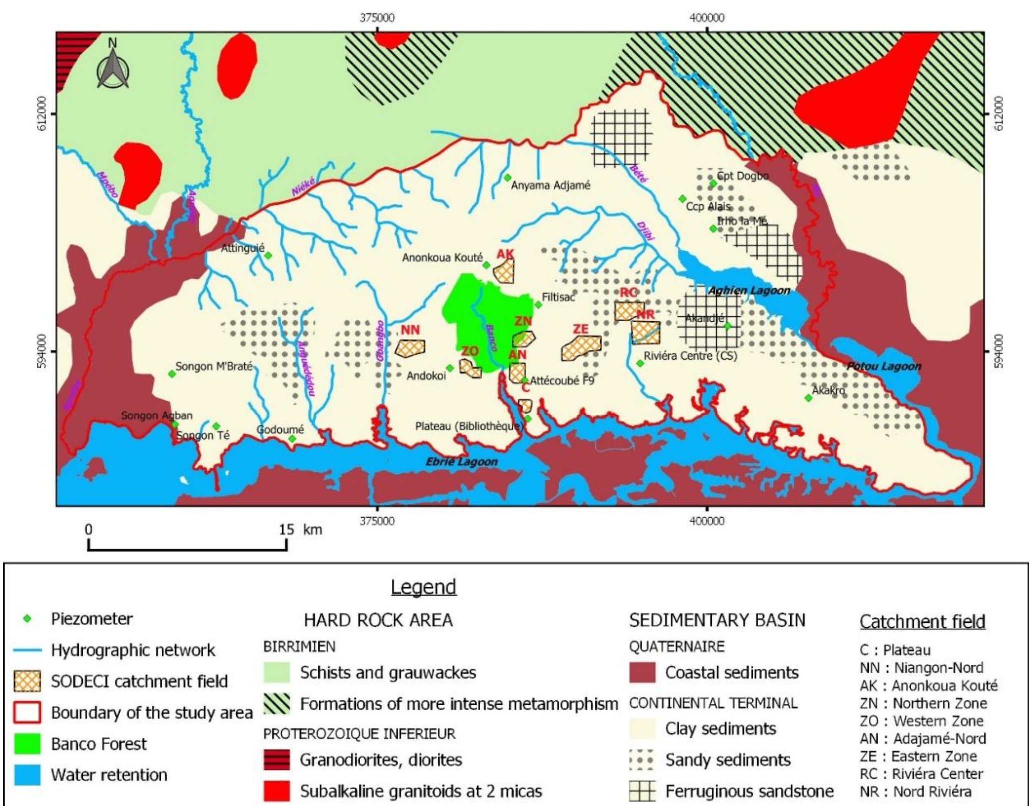

Fig. 1. Location of the study area

The strong increases in Abidjan's water demand (161.99 million m^3 in 2016 and 164.6 million m^3 in 2017, [4] with very little recharge have put a severe strain on the groundwater in the Continental Terminal aquifer.

Faced with the demographic pressure that Abidjan is facing and the growing need for water of its population, we questioned the capacity of the Abidjan groundwater to produce enough water to satisfy it. This question is answered by measuring transmissivity over the entire Continental Terminal aquifer in order to establish an emergency plan and sustainable groundwater management. The transmissivity of aquifer systems is the main hydraulic parameter that controls long-term withdrawals and is therefore important for the development and management of groundwater exploitation. Especially in heterogeneous hydrogeological environments, small variations in small-scale transmissivity can limit local pumping rates and groundwater production [5]. There is no doubt that the most reliable method for estimating the transmissivity of an aquifer is to perform a point pumping test analysis [6]. However, this direct approach to measuring transmissivity over the entire groundwater table requires enormous technical and financial resources that are technically and economically impossible to satisfy [7].

In such cases, hydrodynamic modelling using an inverse approach is the appropriate solution. First, it allows you to bypass the scale change problem. Then, it avoids measuring transmissivity everywhere on the domain, but allows us, using some observations of the state of the system and some control variables, to determine this hydrodynamic parameter over the entire groundwater table [8, 9,10].

The objective of this work is to estimate a transmissivity field over the entire aquifer of the Continental Terminal of Abidjan, thus providing information on the availability and abstraction of water resources in this aquifer using a numerical model based on the Hydrology Pre/ Post-Groundwarter Modelling System (HPP-GMS) code based on the Galerkin finite element method developed around triangular elements.

2. METHODOLOGIES

Hydrogeological problems are flow problems in porous and fissured media whose physics can be very complex and difficult to understand.

In this study, the determination of the known transmissivity field of the Continental Terminal aquifer was built around three main axes: the physical characterization of the aquifer (conceptual model), the conceptualization of the numerical model of the aquifer (meshing, input data, boundary conditions, etc.) and the search for the structure and dimension closest to the natural environment following a series of head simulations by the inverse multi-scale parametrization method. In other words, this theoretical and simplified representation of the Continental Terminal aquifer includes the definition of a structure, the formulation of laws connecting the inputs of the system to its outputs and the definition of the parameters entering into these relations [11]. Although there are many methodologies for the design of a hydrogeological model [12,13,14], they all follow the same general pattern described in the following Fig. 2 according to this model of the Continental aquifer of Abidjan.

2.1 Physical Characterization of the Continental Terminal Aquifer

The physical characterization of the Continental Terminal aquifer was summarized in conceptual models and established operating assumptions.

The Continental Terminal aquifer has a two-layer structure whose total thickness varies between 30 and 160 m in the east-west direction and also varies between 30 and 120 m in the north-south direction (Fig. 3).

The superior layer represents fine to medium sands with a maximum thickness of 80 m. The lower layer of coarse sand has a maximum thickness of 90 m. This set forms only one free aquifer. The roof of the Continental Terminal aquifer, which is made up of the boundary between sandy-clay and fine to medium sandy formations, has an GLCI (General Level of Côte d'Ivoire) rating of between 0 and 100 m. The aquifer wall is the contact between the coarse sand and the basement and its dimension is between 30 and -160 m (GLCI).

With regard to hydrodynamic parameters, the point transmissivity values deduced from the pumping tests differ from one point to another. The highest transmissivity of $3.37.10^{-1}$ m^2 s^{-1} and the lowest one of $2.77.10^{-2}$ m^2 s^{-1} were determined in the catchement field of Water Distribution Company of Côte d'Ivoire named North Zone respectively at the borehole North Zone 12 and North Zone 14 (Fig. 1).

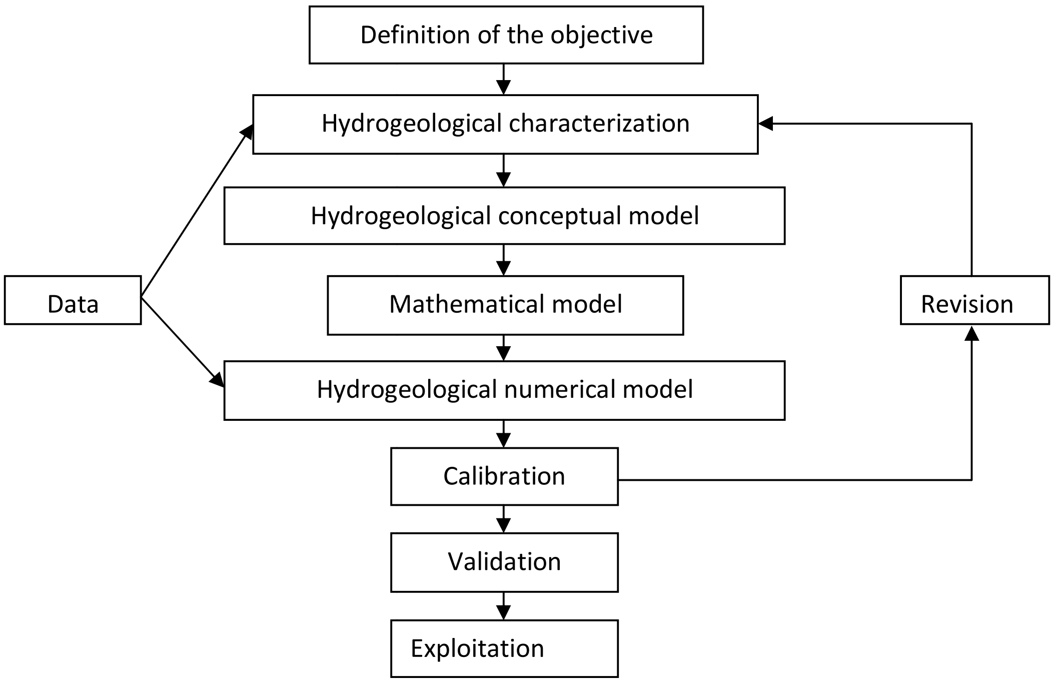

Fig. 2. Hydrogeological modeling process Steps (JANSSENS CORON, 2007)

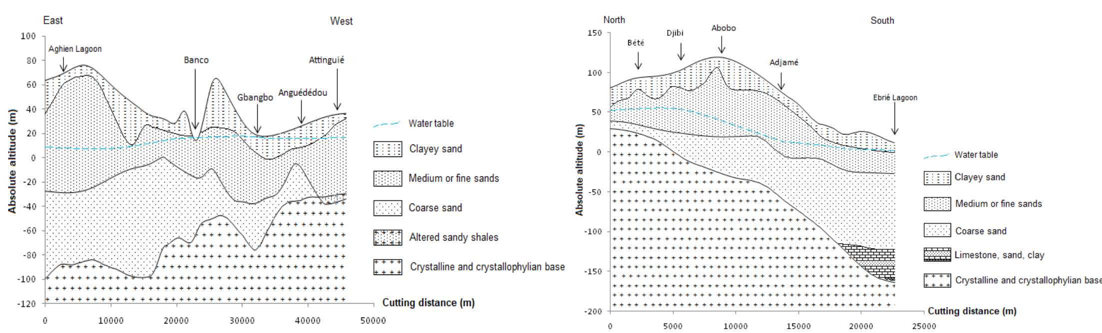

Fig. 3. East-West and North-South hydrogeological conceptual model of the Continental Terminal

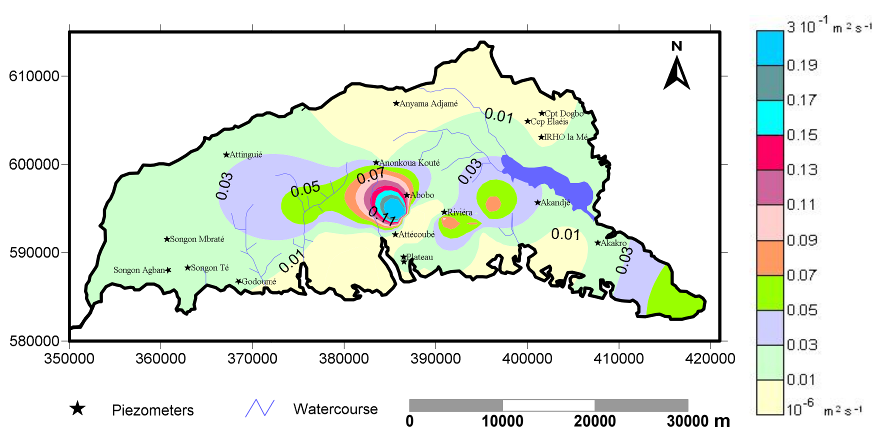

Fig. 4. Interpolation of transmissivities of the Continental Terminal aquifer

These two boreholes belong to the same catchment field. The spatial distribution of the five (5) transmissivity values determined in this work and the forty-three (43) other transmissivity values defined in horizons n3 and n4 of the Continental Terminal aquifer by authors such as Loroux [15], Aghui and Biémi [3], Jourda [16] and SOGREAH [17] is shown in Fig. 4. The transmissivity values in the study area range from 10^{-6} to 10^{-1} m^2 s^{-1}. These transmissivities are few in number and poorly distributed on the domain (Fig. 4) [18,19]. Most of the groundwater table has transmissivities between 10^{-6} and 7.10^{-2} m^2 s^{-1}. The highest values in the range of 10^{-1} do not extend over large areas and are almost all clustered in the North Zone catchment field area. After Adjamé, Zone Nord, the fields collecting from the Riviéra have the highest transmissivity values between 5.10^{-2} and 9.10^{-2} m^2s^{-1}.

2.2 Digital Model of the Continental Terminal Aquifer

The development of the numerical model began with the spatial discretization of the domain after the definition of the boundaries of the Continental Terminal groundwater domain in Abidjan. Fig. 5 shows the numerical conceptualization of the aquifer model of the Continental Terminal of Abidjan in finite element meshing. This mesh of triangular meshes has fixed points for observations, sampling, exchanges and boundary conditions. Thus, we observe that piezometers, catchment fields, stream points selected to take into account groundwater exchange and boundary conditions are located at the nodes (Fig. 5). The discretized study area has 534 meshes for 320 nodes.

The boundary conditions are summarized as potentials imposed at the North and South borders, zero imposed flows at the West and North-East borders, non-zero imposed flows entering the South-East border. The exchanges between the Continental Terminal groundwater and these rivers are only taken into account for the Gbangbo, Banco and Bété rivers. These exchanges involve the level of the groundwater table, the rating of the river bottom and the rating of its free surface, and an exchange coefficient between the two environments. Information on rivers was provided by the Sub-Directorate of Hydrology through the various gauging stations located on these rivers. The exchange coefficient, which is in fact a velocity, has been assimilated to the permeability of the ground on the bottom of the river.

Thus, from the lithological map of the unsaturated zone of the Continental Terminal (groundwater-river exchange zone), the geological nature of the bottom of rivers over all their lengths was determined. On the basis of the permeability of the Continental Terminal's

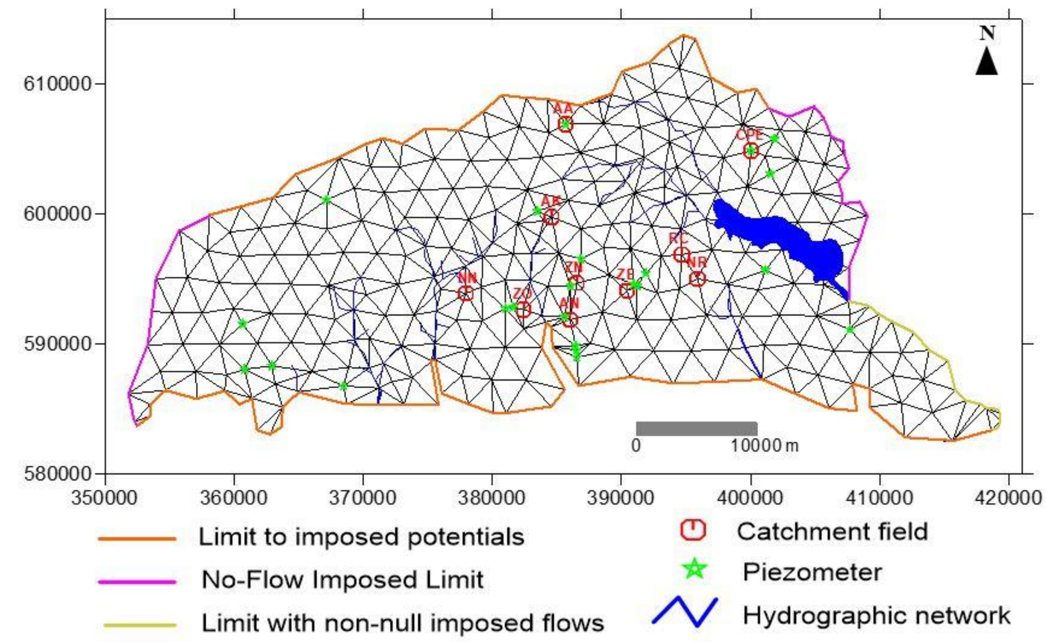

Fig. 5. Conceptualization of the Continental Terminal aquifer numerical model

geological formations according to their nature [20], the necessary exchange coefficient has been defined along each river. The Agnéby, Anguédédou, Mé and Djibi rivers and the Aghian and Potou lagoons are not taken into account in the groundwater-river exchanges because the Sub-Directorate of Hydrology has no data on these rivers. Determining boundary conditions in modeling is a very delicate task and can be a source of error input that will affect all calibration results. This data can therefore be set in turn to force the parameters to remain within a range of permissible values.

Piezometric data are provided by measurements taken on 24 piezometers distributed over the Continental Terminal groundwater. The piezometry values was retained over a period of 1 year (May 2005 - April 2006) divided into 12 monthly time steps for a transient calibration. The demands on the groundwater are essentially limited to the daily production of the 71 functional wells of the nine (9) SODECI (Water Distribution Company of Côte d'Ivoire) catchment fields in Abidjan, with the exception of the wells of the plateau capture field, which is closed. The flows taken from a SODECI borehole in Anyama Adjamé and another irrigation borehole on the Elaeis camp farm were also taken into account. These flows are considered negative, since they come out of the groundwater. The debits considered always concern the period from May 2005 to April 2006 at monthly time intervals. The recharge is considered uniform and constant at each time step over the entire Abidjan nappe, but varies from time step to time.

2.3 Estimation of the Transmissivity Field

By always considering the equation of flow in a saturated porous medium (diffusivity equation), the estimation of transmissivities can be summarized as the minimization of the objective function on loads and transmissivities by calculating their gradient using the adjoint state method. The optimal multi-scale parametrization is also used in the estimation of transmissivities. Transmissivity measurements are the cornerstone of our proposed method. The ideal would be to have reliable spot measurements, in sufficient quantities and well distributed over the area. The interest of this distribution is to first orient the parametrization of the model by bringing out an image of the structure of the real environment. Then, the measured values of the transmissivities are used to calculate the a posteriori criteria for selecting the different parametrization meshes associated with the different parameter sets. They are mainly concentrated in the central part of the study area and are therefore not very representative of the entire study area. For a calculation range of 534 meshes, only 40 meshes were assigned a point transmissivity value from a pump test analysis (Fig. 6).

In addition to quantitative information on the groundwater, qualitative information should be taken into account in the construction of models because it can help to avoid the choice of unlikely solutions. Qualitative information may include general flow characteristics and groundwater supplies. The general appearance of the piezometric surface is North-South. The groundwater is mainly supplied to the North at the basement-sediment contact. The Continental Terminal aquifer is very heterogeneous.This information can help to define refinement areas in the parametrization. Such information, even if considered purely qualitative, may avoid retaining a completely outlier that meets the numerical criteria for estimating transmissivities. For this reason, it is no longer desirable to be limited to finding parameter sets that achieve the best match between the loads simulated by the model and the measured loads. The estimated sets of transmissivities must also be conditioned by the local values of these transmissivities measured in the field. In the event that the measurements are insufficient to characterize the reliability of the estimated transmissivities, qualitative information can be used to constrain the results within plausible limits.

2.3.1 Multi-scale parametrization

There are many inverse methods for estimating the parameters that control the groundwater flow system. Mayer and Huang [21] used the maximum likelihood estimation method to identify parameters with determined variables and parameter residues. Tsai and Yeh [22] proposed a generalized parametrization method and a Bayesian estimation for characterization and identification of parameter heterogeneity in groundwater modelling.

Shih-Ching Wu et al. [23] used the Tabu search algorithm and the adjoint state method to identify the transmissivity components of homogeneous and heterogeneous anisotropic aquifers.

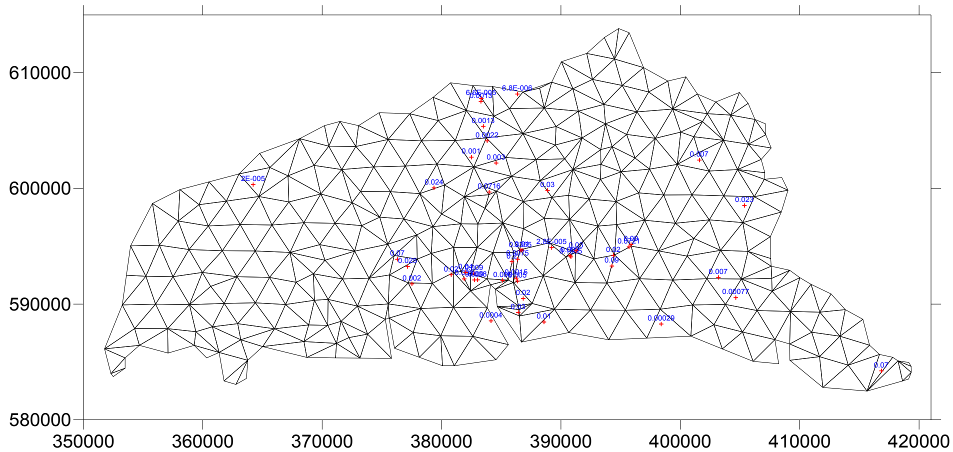

Fig. 6. Spatial distribution of transmission measurement points and associated values (m^2/s)

To represent the natural environment, numerical methods discretize the spatial domain by generating several thousand elements. For a heterogeneous aquifer, each of these elements can be assigned a transmissivity value. The inverse problem therefore has several thousand unknowns (or degrees of freedom) considered as the adjustable parameters of the system. Knowing that the number of observations concerning the dependent variable (ie hydraulic head) is much smaller (of the order of one or more tens), it is clear that the number of unknowns is far too high and must be imperatively to be reduced. Such a case is similar to solving a system of equations where the number of unknowns far exceeds the number of equations [24]. The identification of a very large number of parameters resulting from the discretization of the problem, based on some field observations on the state of the system, is an ill-posed problem. The opposite problem is wrong because it does not have a unique and stable solution and the number of measurements is insufficient. Two possibilities exist to bring the two quantities to the same level. The first solution, which is to increase the number of measurements is not feasible for economic reasons. The second, more accessible, is to reduce the number of parameters to identify. The technique for reducing the number of parameters of a model is the parametrization of the model [25].

Parametrization consists in reducing the number of parameters to be identified by the inverse approach. The parametrization makes the problem more stable, which reduces the difficulties of convergence and makes the problem "better posed". Two types of errors are associated with the inverse problem: the modeling error and the error associated with the uncertainty of the parameters. An increase in the number of parameters will generally reduce the model error, but will increase the uncertainty of the parameters used and vice versa [10]. The optimal level of parametrization therefore depends on the quantity and the quality of the observations. In general, to obtain a valid solution, the number of adjustable parameters must be less than the number of observations. Different types of parametrization are proposed in the literature, including zonation, geology, interpolation, multi-scale procedure.

As part of this work to determine the real transmissivity field of the Continental Terminal, it is the non-automatic multiscale parametrization method that better appropriate the modeling assumptions and will be adopted.

The idea of non-automatic multiscale parametrization is to use several different structures from the start to generate a set of possible solutions. This approach consists in representing the structure of the environment by more and more elaborate structures, during the identification of the parameters of the model.

The non-automatic multiscale identification procedure starts with a large mesh, consisting of four meshes and five nodes covering the entire computational domain (Ackerer et al., 1996). After this first step, follow directly the series of meshes to be tested (Fig. 7).

This approach consists in representing the structure of the environment through increasingly elaborate structures during the identification of the model parameters. This results in an increase in the size of the parametrization model, by refining the mesh from one step to the next [26]. It is therefore in this state of mind that the identification of the transmissivities of the Abidjan groundwater model was carried out around eight basic structures called structure 100 to structure 800 (Fig. 8).

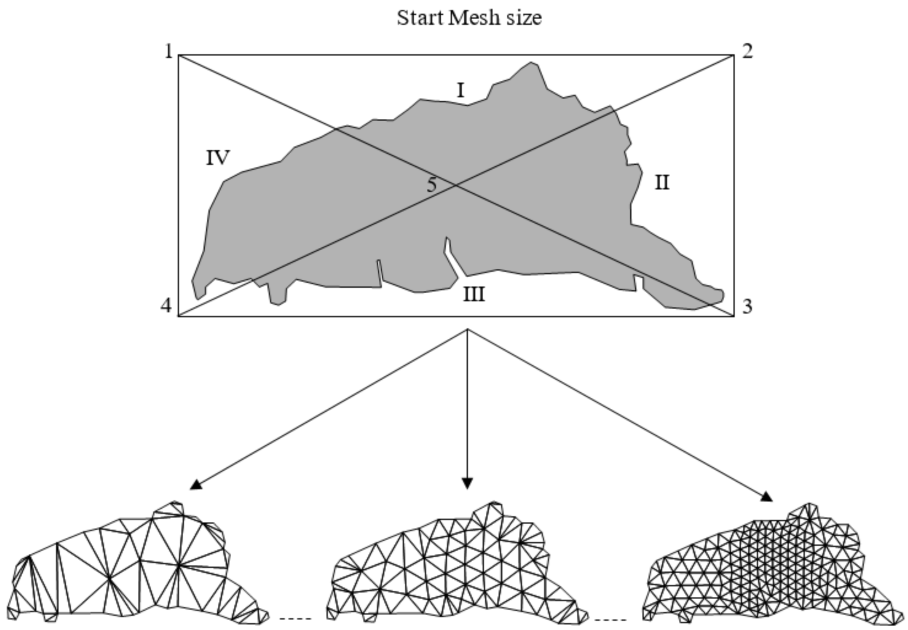

Fig. 7. Procedure for non-automatic multi-scale parametrization

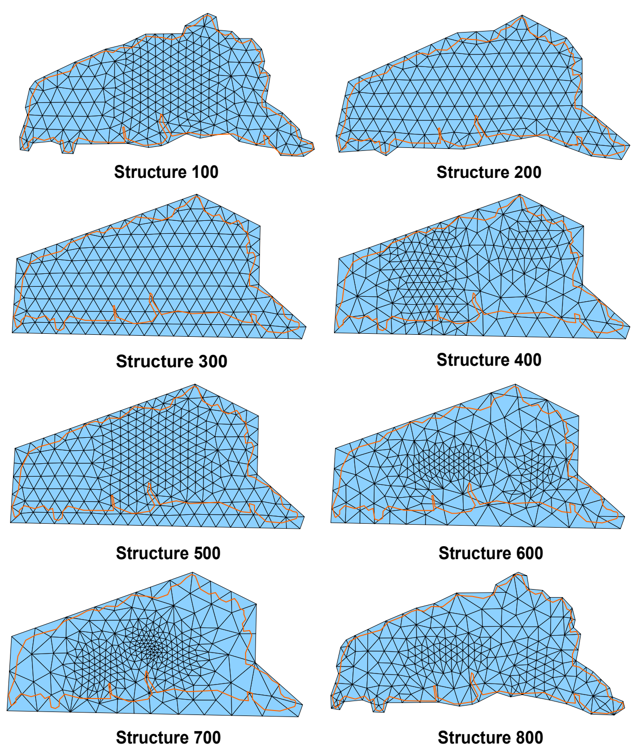

Fig. 8. Representation of the eight structures tested

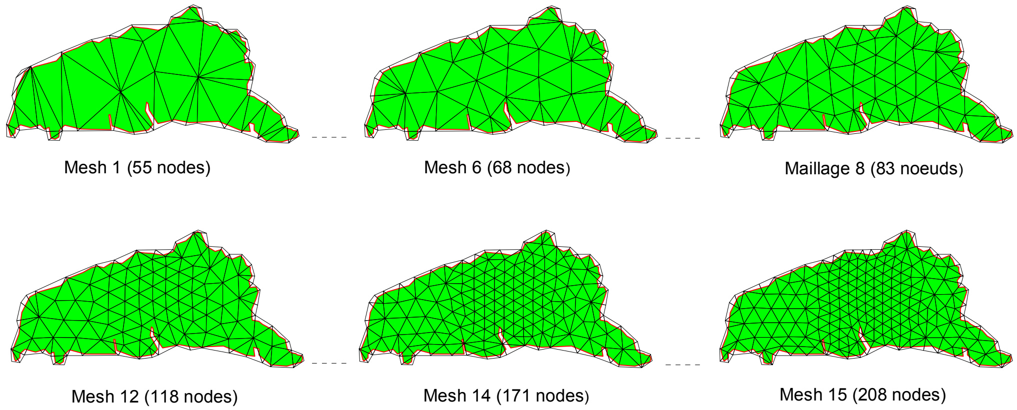

Fig. 9. Evolution of the dimension of the parametrization meshes of Structure nº100

The proposal of these initial structures was guided by the simultaneous exploitation of several information on the physical environment described in this study (point transmitsivities, specific flows, geological data, land use, etc.), without forgetting the general knowledge we have of the sector based on studies already carried out [20,5,27,3,16,17,26]. Thus, with the help of this information, we have refined or not certain areas of the study area to generate the different structures.

Each structure is then made up of a series of parametrization meshes of gradually increasing size to be tested (Fig. 9). The total number of parametrization meshes to be tested of the eight structures combined is 115 meshes.

2.3.2 Model calibration and validation

The purpose of model calibration is to reproduce the hydrodynamic functioning of the groundwater as closely as possible by adjusting the various parameters that we believe can be varied while remaining within the range of realistic values [28]. To improve the simulation results, in addition to transmissivity, the groundwater exchange coefficients, flows, free river surfaces and porosity were adjusted.

Once the series of meshes is available, the model is successively calibrated with all the meshes contained in the series. But before starting the calibration, the lower and upper bounds of the model's conceptualization parameters are determined according to our knowledge of the study environment. Once the calibration is started, it is observed that as the number of parameters increases, the objective function decreases. The objective function decreases very quickly at the beginning and stabilizes around a value above a certain dimension (number of parameters) of the parametrization. All the parametrization meshes that gave these satisfactory results on the value of the objective function can be used as possible approximations of the structure of the studied environment [7]. In this case, the only criterion used is the value of the objective function. A multitude of solutions would then be obtained for a problem that in reality has only one since the actual structure is unique. To limit the effects of the inevitable non-uniqueness of the solution of the opposite problem, it is therefore essential to find a criterion other than the objective function. This criterion will be used to evaluate the quality of the parameter set associated with each parametrization model and to reduce the number of solutions to the opposite problem. To assess the quality of each parametrization mesh, two performance criteria are determined.

The first criterion is the classic load criterion, which indicates to what extent the model was able to simulate the state of the system on the basis of available data. For language convenience, the reference is to this criterion as model error, which is noted as Error_H in the rest of our discussion (**Eq. 1**).

$$\text{E}rror\_H = 100 \cdot \sqrt{J(P)/N \times Np} \quad (1)$$

with:

$$J(P) = \sum_{n=0}^{N-1} \left\| H_c^{n+1} - H_m^{n+1} \right\|^2$$

H_c : Vector of loads simulated by the model at observation points.
H_m : Vector of loads measured in the field at observation points.
N : Number of time steps
Np : Number of load observation points in the field.

Error_H represents the average over all measuring points and at all time steps, of the difference between the simulated loads and the measured loads.

The second criterion concerns the quality of the estimated parameters with the parametrization mesh used for calibration. We refer to this criterion as a structural error, which we note as Error_P with reference to the parameters (Eq. 2). The structural error is defined from local parameter measurements from pumping tests.

$$Error\_P = 100 \cdot \sqrt{\frac{1}{M} \sum_{j=1}^{M} \left(\frac{Pc_j - Pm_j}{Pm_j}\right)^2} \qquad (2)$$

Error_P : average of the relative deviations of the identified parameters from the measured parameters.

Pc_j : Parameters identified in point j.
Pm_j : Parameters measured at point j.
M : Number of measuring points of the parameter P.
M represents, for example, the number of meshes in the calculation model where a parameter value is available.

This can be done for any information a priori on all the other parameters searched.

In this modelling of the Abidjan nappe, transmissivity is the desired parameter. The criteria *Error_H* and *Error_P* are then adapted for the circumstance.

The model error *Error_H* retains its original shape and therefore represents the mean of the quadratic deviations between the loads simulated by the model and the loads measured at piezometers over the 12 time steps. *Error_P*, which represents the mean of the relative quadratic deviations between the identified and known transmissivities, becomes *Error_T* and takes the following form (Eq. 3):

$$Error\_T = 100 \cdot \sqrt{\frac{1}{Ne} \sum_{j=1}^{Ne} \left(\frac{Tc_j - Tm_j}{Tm_j}\right)^2} \qquad (3)$$

with:

Tc_j : Transmissivity identified on element *j* of the calculation mesh.
Tm_j : Transmissivity of the known field strength for the element j.
Ne :Total number of elements of the calculation mesh.

As soon as the successive calibrations are carried out, we have for each parametrization mesh a value of Error_H and an Error_T value, defined from equations (2) and (3).

These two criteria *Error_H* and *Error_T* are then represented in the same graph according to the size of the parametrization (number of nodes of the multi-scale mesh). The analysis of this graph makes it possible to evaluate the quality of the set of parameters associated with each parametrization mesh and, if possible, to select one of them to represent the structure of the environment. The representation of *Error_H* and *Error_T* as a function of the number of nodes of the multi-scale mesh thus makes it possible to select the optimal size of the parametrization model that is suitable for model calibration [7]. The determination of the structure and size of the parametrization must be the two inseparable supports of the identification of the parameters of the inverse models. However, there are no formal methods to identify them. The search for structure and size will then be oriented according to the results of the calibration and the information available a priori.

In practice, a series of different structures are used to generate a set of solutions, some of which are selected according to the behaviour of the *Error_H* and *Error_T* curves.

The overall analysis of the *Error_H* and *Error_T* curves as a function of the number of parameters in the calibration series did not allow to determine a single structure and dimension close to the actual structure of the slick. Then a local examination of the *Error_H* and *Error_T* curves from the calibrations is carried out at the level of the stable *Error_H* value range which constitutes the first solutions of the process of identifying the transmissivity field of the Abidjan nappe.

Then we will link the information collected to the results of the comparison between the simulated loads and the observed loads. The comparison of simulated loads with observed loads was carried out over four months of the year (January, June, August and October), each belonging to one of the four seasons of the year in Côte d'Ivoire, but we will only present the results for the month of June.

To further reduce the number of transmissivity identification solutions and also serve as a validation factor for the identified transmissivity field, transmissivity values that were not used in the implementation of the model will be positioned on the likely transmissivity solutions fields of the opposite problem. These transmissivity values to be positioned were measured using the long-term pumping tests carried out in July 2012 on nine (9) boreholes of the new Anguédou catchment field (Table 1). The objective of superimposing known transmissivities on calculated transmissivities is to search for the identified transmissivity field that best reproduces the actual values of the transmissivities of the Abidjan nappe.

3. RESULTS AND DISCUSSION

3.1 Identification of the Transmissivity Field

The representation of *Error_H* and *Error_T* from the different settings made according to the number of nodes of the multi-scale mesh (Fig. 10) enables this research work to better study the behavior of both types of errors.

It is observed that E_H Model Error, decreases rapidly with the increasing number of parameters and then becomes almost stable despite the increase in the number of multi-scale nodes. The E_H curve allows to divide all the parametrization meshes into two groups. The first group with fewer than 100 nodes fails to match the model parameters to the loads provided. On the other hand, the meshes of the second group, with a number of nodes greater than 100, manage to minimize the differences between the loads simulated by the model and the observed loads. Then each of the meshes of the stable zone of E_H constitutes a solution to the problem which is in reality unique. The analysis of the second criterion, Structural Error or E_T will permit to significantly reduce the number of solutions according to E_H and even to probably choose a mesh from these solution meshes to represent the structure of the model. Thus E_T evolves in general in an oscillatory movement between several extremes. It goes from a minimum to a maximum and vice versa over the entire curve. For small numbers of multi-scale nodes (less than 100 nodes), there are minimums that could reflect the under-parametrization of the model or even be interpreted as a solution if we lose sight of the limited number of transmissivity measurements from which E_T was calculated. The values of *Error_H* and *Error_T* are generally high with a maximum of about 10^6. The fragment of the E_T curve framed in Fig. 10 in the area of the E_H curve that could calibrate the water table, which hightlifht three meshes of parametization selected a priori to identify the model parameters. These are the 105 meshes; 153 and 223 nodes of structure 700, which were chosen because they have low values of *Error_T*.

The three identified transmissivity fields associated with the selected meshes have a good overall structure and are oriented in the direction of groundwater flow. The estimated transmissivity values range from $5.4.10^{-5}$ to 1 m^2s^{-1} and seem very high in some areas.

Table 1. Characteristics of drilling in the Anguédou catchment field

Identification Number	X	Y	Z	Transmissivity (m^2/s)	Pumping flow (m^3/h)
F.2	373617.58	593666.32	157.165	$9.20.10^{-3}$	200
F.3	373452.97	593523.03	155.185	$1.10\ .10^{-2}$	170
F.5	373412.74	593344.64	158.486	$6.10.10^{-2}$	210
F.6	374077.06	593504.25	140.71	$1.44.10^{-2}$	240
F.8	373470.08	593661.34	158.895	$2.20.10^{-2}$	240
F.9	374363.24	593648.88	136.851	$7.50.10^{-3}$	220
F.10	374201.66	593834.45	154.846	$6.40.10^{-2}$	250
F.11	374018.07	593778.57	145.532	$1.20.10^{-2}$	260
F.12	373845.38	593985.79	159.841	$2.0.10^{-2}$	220

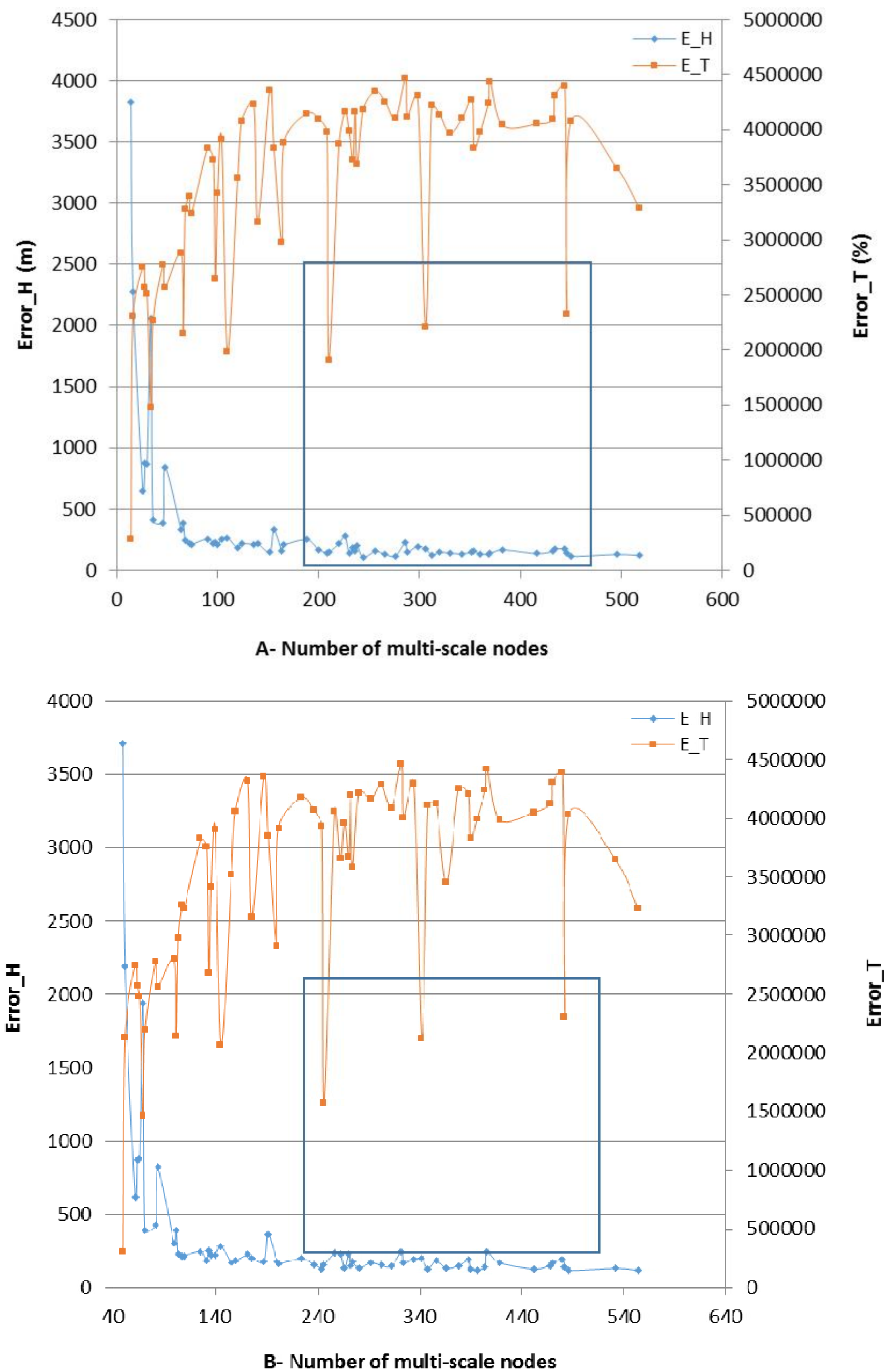

Fig. 10. Representative curves of *Error_H* and *Error_T* as a function of the number of multi-scale nodes: A) Transmissivity, fixed porosity; B) Transmissivity, porosity and fixed exchange coefficient

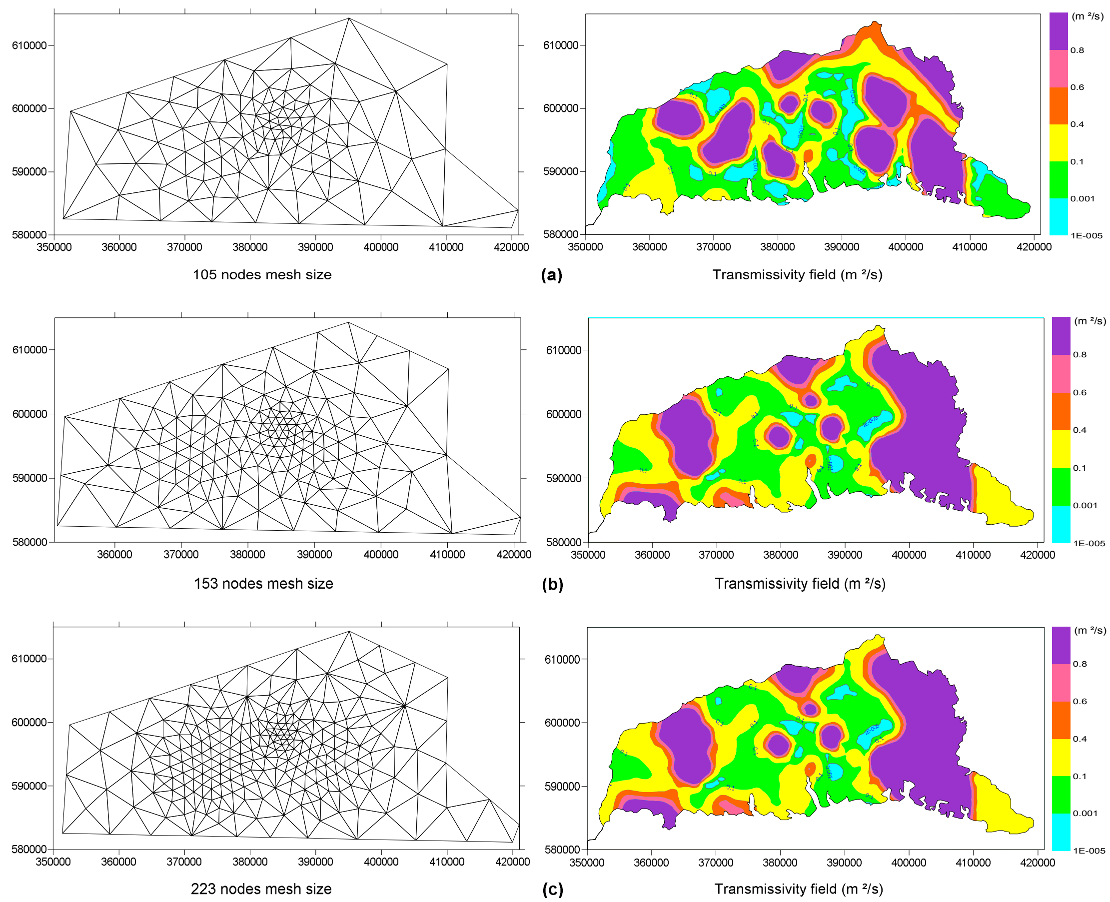

Fig. 11. Parametrization meshes and associated transmissivity field: a) 105 nodes mesh and its transmissivity field; b) 153 nodes mesh and its transmissivity field; c) 223 nodes mesh and its transmissivity field

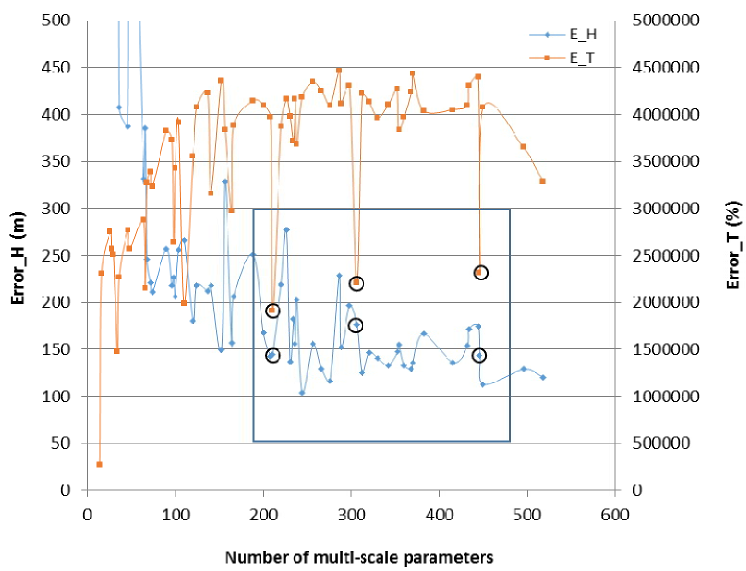

Fig. 12. Local representation of the stable *Error_H* and *Error_T* fraction as a function of the number of multi-scale parameters

The transmissivity field of the 105 nodes mesh has a much wider range of transmissivities between 10^{-5} and 10^{-3} m^2s^{-1} than on the 153-223 nodes mesh fields. In return, the transmissivity classes between 10^{-3} and 1 m^2s^{-1} represent much larger areas on the 153 and 223 nodes mesh fields than the 105 nodes mesh field. High transmissivities are mainly observed in the extreme East and West and in the centre of the three identified fields. However, it should be noted that the 153 and 223 nodes mesh sizes have almost identical transmissivity fields and have two distinct high transmissivity zones at the eastern and western extremes.

Among these three pre-selected meshes (Fig. 11), one of them cannot yet chosen to represent the model parameters because the minimum E_H is not clearly visible and also additional qualitative information is needed.

3.2 Determination of the Optimal Structure and Size of the Model

The analysis of the Error_H curves made it possible to initially have a set of possible solution meshes for the opposite problem. Subsequently, the exploitation of Error_T as a function of the number of parameters led to reducing this set of solutions to three meshes close to the true structure (Fig.10). However, it has so far been impossible to determine a single structure and size close to the actual groundwater structure among these three meshes. The study will therefore be focused on the detail of the portion of the curve E_H solution of the inverse problem previously identified to hopefully designate one of the structures as the structure closest to the actual structure.

3.2.1 Analysis of E_H values solution of the inverse problem

The detailed analysis of the E_H and E_T criteria (Fig. 12) shows that the 223 nodes mesh has the smallest E_H value, the 105 nodes mesh has the second smallest value and the 153 nodes mesh has the highest E_H model error value. The best E_T Structural Error is assigned to the 105 nodes mesh, then the second best value is assigned to the 153 nodes mesh and finally the lowest value is assigned to the 223 nodes mesh. However, it should be noted that the E_H and E_T values of the 153 nodes mesh are generally very close to those of the 223 nodes mesh. This information on the values of E_H and E_T still does not allow us to make a coherent choice of the true structure among these three meshes.

The detailed analysis of the *Error_H* and *Error_T* curves still does not make it possible to distinguish the three transmission fields determined and to make a single choice. These many different observed minima are probably due to the insufficient number of known parameters and measurement errors in these parameters. In the absence of other criteria for differentiating these indicated meshes, the comparison of the simulated loads associated with each parametrization mesh with the loads measured at the observation points is used.

3.2.2 Comparison of simulated loads to measured loads

The comparison of simulated heads with the loads measured in June 2005 is shown in Fig. 13. Overall, the three parametrization meshes identified show that the simulation almost accurately reflects the general flow direction of the slick except for a slight change in direction in the banco valley area. The adequacy between the simulated piezometric lines and the measured piezometric lines is acceptable overall. However, it should be noted that the transition from 105 nodes to 153 nodes mesh size improves model simulations because the piezometric lines from the calculated loads are closer to the reference lines (Fig. 13 a, b).

However, the change from 153 to 223 nodes degrades the improvement in the simulation sometimes observed. On the other hand, as the mesh size of the structure increases to 223 nodes, it becomes apparent that the calculated loads are moving away from those determined at 153 nodes (Fig. 13 b, c). Oscillations of the calculated piezometric curves are visible around the observed piezometric curves. The comparison of simulated loads with the loads observed in this work in January, August and October is similar to that of June 2005 presented above. The analysis in Fig. 13 shows that the 153-nodes mesh best reflects the observed head.

The comparison of observed and calculated loads determined the 153 nodes size of structure 700 as the optimal size of the Continental Terminal groundwater model. To support this choice, a classification of the characteristics of the different parametrization meshes selected is presented in Table 2.

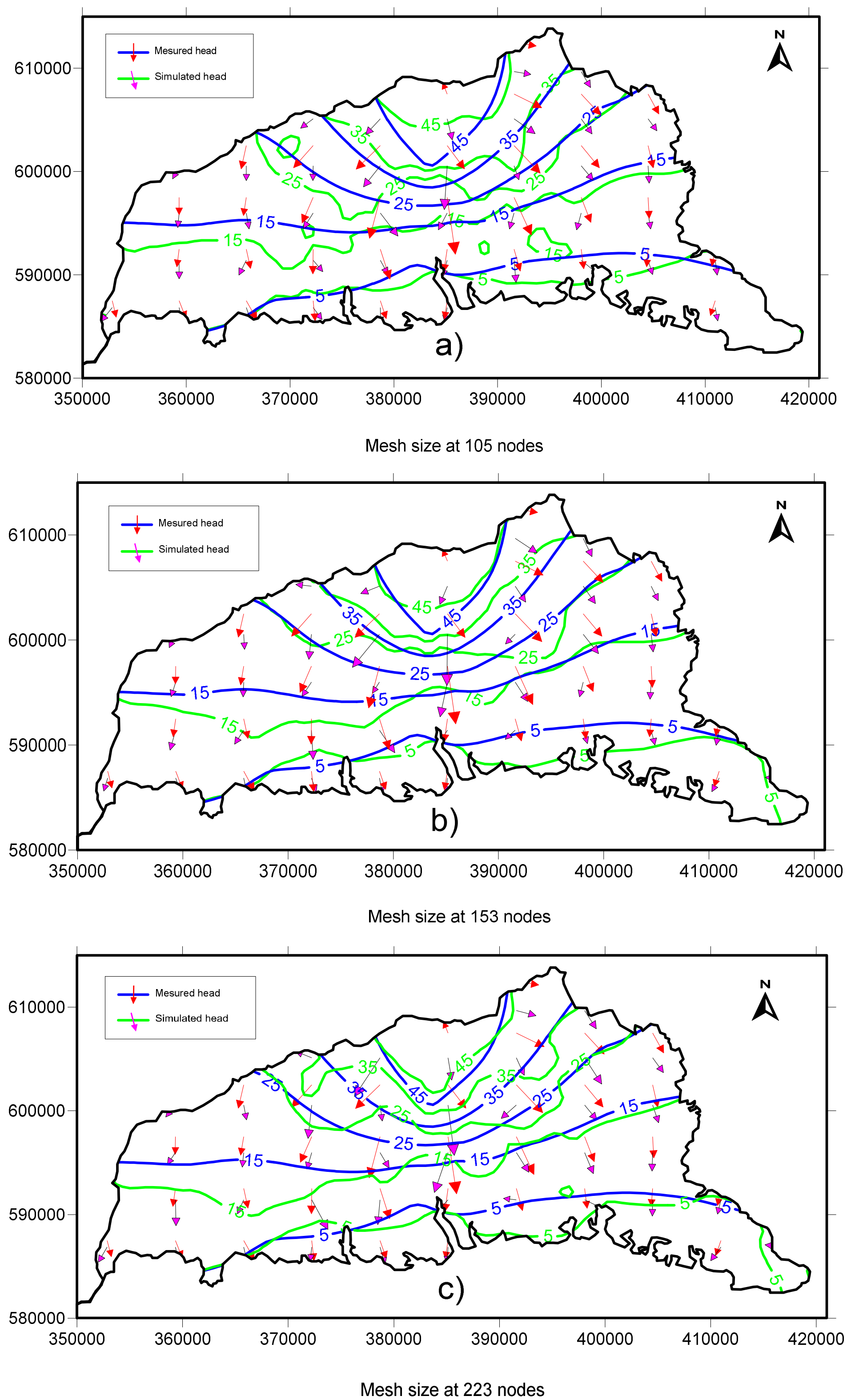

Fig. 13. Comparison between the measuring head and the simulated head in the three meshes selected in June 2005: a) 105 nodes mesh; b) 153 nodes mesh; c) 223 nodes mesh

Table 2. Classification of the characteristics of the different parametrization meshes selected

N° of Structure	Mesh size nodes number	Error_H	Error_T	Simulated piezometry
700	105	2e	1er	3e
700	**153**	3e	2e	**1er**
700	223	1er	3e	2e

The three meshes selected belong to structure 700. The 105 nodes mesh has the best E_T values. The 153 nodes mesh has the best piezometric restitutions. The 223 nodes mesh in turn has the best results of E_H.

The ranks of E_H and E_T are taken into account with less importance given the insufficient quantity and quality of the measures previously demonstrated. Indeed, several minima have been observed in Fig. 10. This reflects the measurement errors and justifies the lower importance given to the E_H and E_T ranks at the expense of the ranks occupied by the simulated piezometry. This selection criterion is far from objective, however, it is a way of taking qualitative information into account. The first rank occupied by the quality of the piezometry simulated by the 153 nodes mesh is then more expressive than the first rank of the 105 nodes mesh occupied for the E_T values and even more important than the first rank of the 223 nodes mesh occupied for the E_H values.

Presumably, from the analysis of the rankings of the classification of the characteristics of the different parametrization meshes selected from Table 2, it can be deduced that the identification of the parameters by inverse approach does not solve the problem of model calibration but allows a more relevant analysis of the results in the modelling by highlighting gaps and/or inconsistencies in the available information and contradictions in the assumptions made when the model was constructed.

At the end of our investigations, we propose the transmissivity field associated with the 153 nodes mesh as the structure closest to the actual aquifer structure of the Continental Terminal in Abidjan (Fig.14).

3.2.3 Validation of identified transmissivity fields

Fig. 15 shows the position of the Anguedou catchment field on the identified transmissivity field. This wellfield consists of nine (9) boreholes that appear on the map as a single cross because they are all located within a radius of less than one square kilometer (1 km^2). The measured transmissivity values of the nine (9) positioned boreholes are all within the ranges of 10^{-3} and 10^{-2} m^2/s.

The measured transmissivities of the Anguédédou catchment field belonging to the ranges of 10^{-3} and 10^{-2} m^2/s are located in the transmissivity class of 10^{-3} to 10^{-1} m^2/s of the transmissivity field identified at the end of the simulations. At the end of this validation test, it could therefore be deduced that the transmissivity field from the 153 nodes porosity mesh in Fig.16 below best represents the true structure of the Continental Terminal aquifer in Abidjan.

3.3 Discussion

The modelling of the Continental Terminal groundwater flows was carried out around eight starting structures. Several structures are tested to indicate the inevitable subjectivity in the realization of the basic structures and the interpretation of the available data.

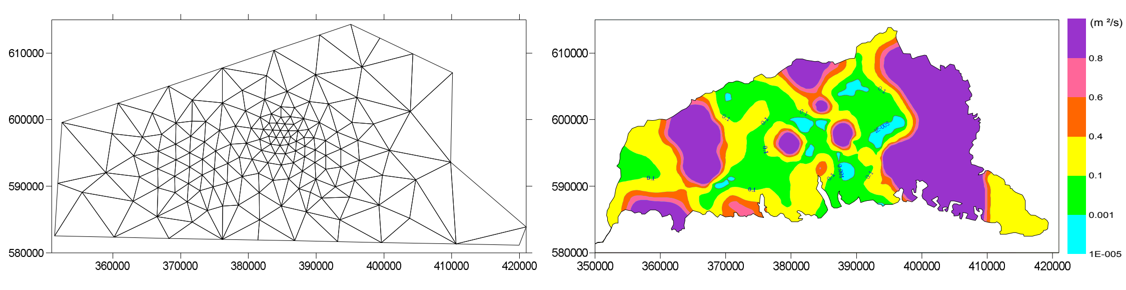

Fig. 14. Selected transmissivity fields from the 153 nodes mesh

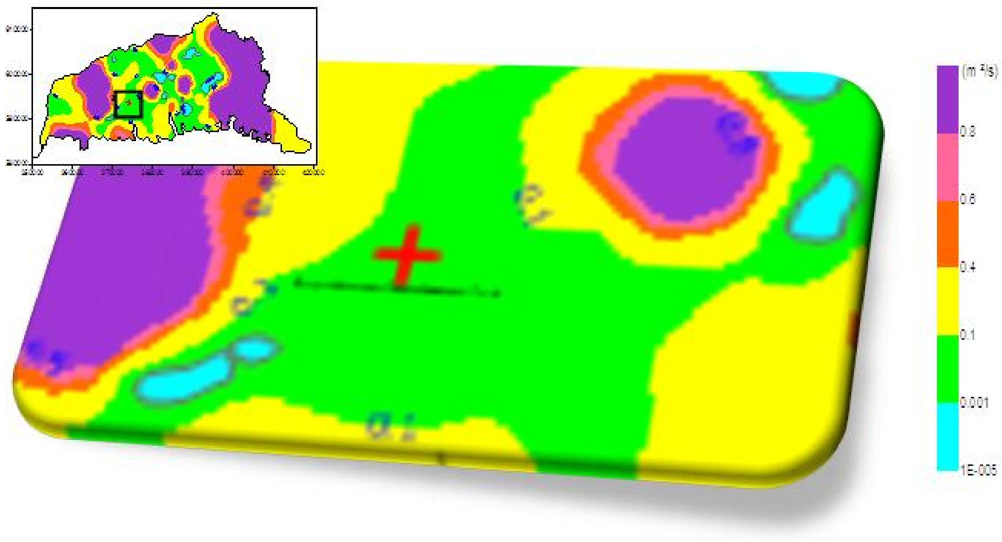

Fig. 15. Positioning of the Anguédédou catchment field on the identified transmissivity field

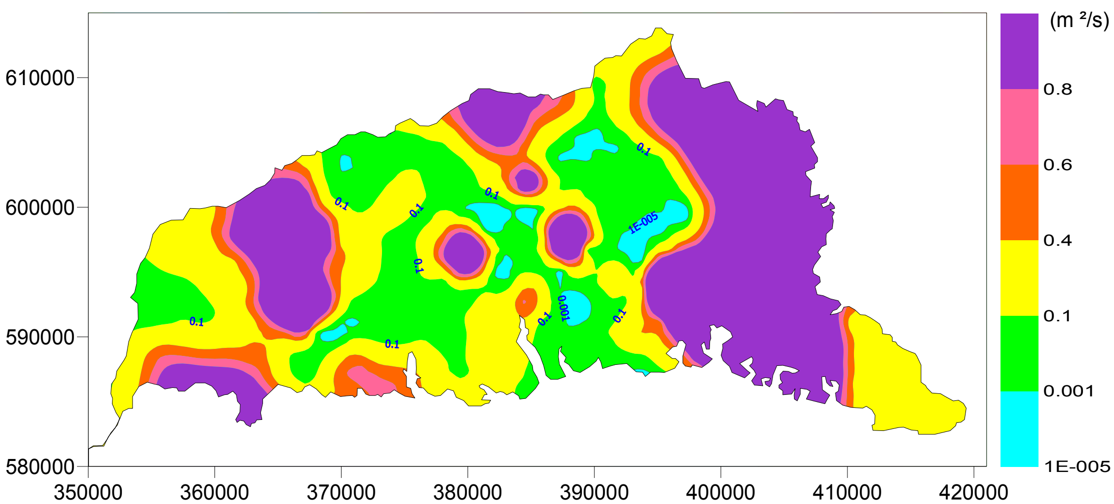

Fig. 16. Transmissivity field identified as the structure closest to the aquifer structure of the Continental Terminal in Abidjan

The analysis of the Error_H and Error_T criteria following the calibration series made it possible to determine the 105, 153 and 223 nodes mesh sizes of structure 700 as solutions to the opposite problem. As it is very often the case in many fields of geosciences, the study is typically faced with a poorly posed reverse problem because it does not admit a single and stable solution. Therefore, the only way to reduce these uncertainties would be to add information either in the form of observed piezometries or in the form of pump tests. The values of the objective functions E_H and E_T between 0 and 5.10^6 remained high, as in the work of Lehmann 1997, where the objective function O(p) sometimes went as high as 109. These objective function values are said to be high compared to the near-zero values of objective functions obtained in theoretical studies that are generally conducted on simple cases and with data under control. The reasons for the high objective functions are probably related to the complexity of the natural study environment and the insufficient number of actual measurement points. According to De Linares [29], this lack of measurement is a problem for the calibration and validation of the model as observed in this work through the high values of E_H and E_T obtained.

The boundary conditions and groundwater stresses initially introduced into the conceptualization of the model may be the cause. Indeed, in the evaluation of the boundary conditions of the model, the difficulties of quantifying the effects of rivers are mentioned on the groundwater. In the absence of all data, rivers such as Anguédédou, Niéké, Angéby, Djibi and even the Aghian and Potou lagoons were simply ignored. However, the influence of these rivers on the water table has been demonstrated for some [15,17]. Even for the three rivers (Banco, Gbangbo and Bété) taken into account, the number of parameter measuring stations installed on each river according to their length is not exhaustive. The work of Kouassi [7] and Thierion [30] has also been confronted with this problem of quantifying groundwater-river relations.

With regard to the identified transmissivities, the transmissivity fields associated with the three selected meshes have a good overall structure in accordance with the hydrodynamics of the groundwater. However, the spatial distribution of the identified transmissivities (Fig.16) differs significantly from the linear interpolation distribution of the few transmissivity data measured in the field (Fig. 4). This clear difference between the identified and interpolated transmissivity fields shows that a spatially variable parameter such as transmissivity cannot be determined by a simple linear interpolation of some measured parameters, because the heterogeneity and anisotropy of the natural environment constitute a limit to linear interpolation. Then the transmissivity field resulting from linear interpolation cannot be considered as a solution of the desired field.

To return to the identified transmissivities, values below 10^{-4} m^2s^{-1} correspond to impermeable and semi-permeable zones, i.e. non-productive and low-productive zones. Transmissivities greater than 10^{-4} m^2s^{-1} represent permeable zones that are therefore productive or even very productive [11]. The distribution of transmissivities is also in accordance with the lithological facies of the saturated zone of the Continental Terminal slick determined. Transmissivity values appear very high in some places with transmissivities T> 10^{-1} m^2s^{-1}, because according to MacDonald et al. [31] aquifers with a transmissivity of about 10^{-2} m^2s^{-1} are very productive and therefore highly transmissive aquifers. However, these high transmissivity values above 10^{-1} m^2s^{-1} in the context of the groundwater of the Continental Terminal in Abidjan make sense, as they confirm the information that the transmissivity values of previous studies in the order of 2. 10^{-1} m^2s^{-1} are underestimated values [32]. Indeed, the boreholes are strained only over a small fraction of the thickness of the aquifer for an often short test period; therefore, the transmissivity value that is derived from the pumping test probably constitutes a lower limit of local transmissivity. So the upper limit of 1 m^2s^{-1} of the estimated transmissivities remains a possible order of magnitude.

These estimated transmissivities (5.4 $10^{-5} \leq T < 1$ m^2s^{-1}) are partly within the range of transmissivities measured in several continental or coastal sedimentary aquifers in Africa. These aquifers are generally transboundary. Thus, the transmissivities measured in the coastal sedimentary basin located in southwest Ghana, which is a continuity of the aquifer of the Ivorian Continental Terminal, newly called the Tano basin on either side of the countries, vary between 8.1. 10^{-6} and 1.8. 10^{-5} m^2s^{-1} [33].

As for the Continental Terminal aquifer shared by Togo and Benin, the transmissivities are in the order of 8.10^{-4} and 3.10^{-2} m^2s^{-1} [34]. For the sandstone aquifer of Nubia in northeastern Africa, which is divided between Egypt, Libya, Sudan and Chad, transmissivity ranges from $9.9.10^{-4}$ to $6.9.10^{-2}$ m^2s^{-1} [35]. With regard to the continental sedimentary formations of the Chad Basin encountered in the Central African Republic, Chad, Cameroon, Niger and Nigeria, the transmissivity of the Continental Terminal aquifer in the capital Maiduguri locality of Borno State in Nigeria varies between 3.8. 10^{-4} and 1.2. 10^{-3} m^2s^{-1} [36].

Referring to the history of transmissivities measured over the various African sedimentary basins mentioned above, it can be deduced that transmissivities greater than 10^{-1} m^2s^{-1} are probably overestimated. However, Loroux 1978's work on the Ivorian coastal sedimentary basin states that the transmissivity of the Continental Terminal aquifer is between $1.4.10^{-1}$ and 2. 10^{-1} m^2s^{-1}. Also, studies by Scet Ivoire et al. [32] on the same aquifer at the Continental Terminal show that transmissivity values in the order of 2. 10^{-1} m^2s^{-1} from the interpretation of pumping tests are underestimated values. Then the estimated transmissivity values on the aquifer of the Continental Terminal of Abidjan varying between 10^{-1} and 1 m^2s^{-1} are possible. This demonstration also shows that the Continental Terminal aquifer is powerful and highly productive in some areas.

The calibration series allowed the 153 nodes mesh to be chosen as the optimal size of the model. The transmissivity field associated with this optimal dimension is defined as the structures closest to the true structure of the Continental Terminal aquifer. During this calibration, the change from 105 nodes to 153 nodes mesh improves results and the change from 153 nodes to 223 nodes mesh degrades the improvement in results sometimes observed. These results show that the 105 nodes model is sub-parametered and the 223 nodes model is over-parametered.

The transmissivity field resulting from the adjustment of certain parameters such as porosity improved the simulation results, because in the implementation of the model we fixed a uniform porosity value of 0.15 for the whole slick due to lack of data. The Continental Terminal aquifer is heterogeneous, so porosity cannot be uniform. So, choosing a uniform porosity is a source of error introduction.

This trend towards improvement in the identified transmissivities is also confirmed by comparing simulated loads with observed loads. In this comparison, oscillations of the piezometric curves calculated around the observed piezometric curves are observed. These movements of the simulated piezometric curves are explained by the number of measurement points considered in the observation measurements compared to the number of measurement points in the simulation. Indeed, the interpolation of the measured loads of the groundwater was done around 24 measurement points (piezometers) against 320 measurement points (number of nodes of the initial mesh size) in the interpolation of the estimated loads. Then the interpolation of the simulated piezometry is much more constrained (creating the observed oscillations) than the interpolation of the observed piezometry which has much more freedom. The adequacy between the simulated and observed loads could have been better if the number of measurement points of the two situations were equal or approximately equal.

However, it is important to note that the adjustment of one or more parameters does not necessarily guarantee the improvement of the simulated parameters [8] as observed in the calibration series performed. This observation is also demonstrated in the work of Thierion [30] where the adjustment of storage coefficients of the alluvial aquifer of the Upper Rhine (France) makes it possible to improve the simulation of piezometric levels in some areas while the simulations deteriorate in other areas. Thus, in a parametrization, the optimal size and structure that can be extracted are determined by the quantity, quality and spatial distribution of the available measurements, as demonstrated by the work of Kouassi [7], Castaings [37] and Lehmann [8]. In other words, it is only possible to receive from the model what is given to it.

Consequently, the identified transmissivity field could be used as a guide in the implementation of high flow drilling on the Abidjan groundwater and, in return, the success rate of these installations will make it possible to assess the level of credibility of the identified field.

4. CONCLUSION

The Continental Terminal groundwater flow modelling made it possible to set up a numerical flow model consisting of a mesh size of 534 meshes and 320 nodes. In addition, the identification of the parameters around eight initial structures resulted in the choice of the 700 structure with a size of 153 nodes as the structure closest to the true structure. The transmissivity field associated with this mesh has good overall values. However, these values seem very high in places for sedimentary formations. The adjustment of porosity has made it possible to improve these identified transmissivities while underlining in a very beautiful way that the determination of an optimal structure and size in a model is totally dependent on the quantity and quality of the available data.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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