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Breaking of Spiral Waves Due to Obstacles

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Authors' contributions

This work was carried out in collaboration between all authors. Authors DOL and DLS wrote the first draft of the manuscript. Author RÁP wrote the final manuscript. All authors read and approved the final manuscript.

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Abstract

A spiral wave, which is a self-sustaining wave, is believed to be the source of certain types of arrhythmias, which can lead to fibrillation. In this paper, we study a generic model for the propagation of electrical impulses in cardiac tissue based on the Fitzhugh-Nagumo (FHN) equations. By numerical simulations we consider the evolution of spiral waves and their interaction with obstacles, such as ischemic or dead tissue from a heart attack or surgery. We describe three possible outcomes (attachment, bouncing and break up) when a spiral wave in the trochoidal regime interacts with an obstacle. The results can be useful to understand the dynamics of the interaction between drifting spiral waves and obstacles and to observe that obstacles might act as a switch from a less to a more dangerous arrhythmic regime.

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1 Introduction

Propagation of waves in excitable media is a topic that has attracted the attention of many scientists in the last thirty years [1, 2]. It has important applications in the understanding of the evolution of chemical reactions [3, 4], spreading depression waves in the central nervous system [5] and cardiac arrhythmias [6]. In the later case, waves of excitation propagate through the heart tissue, giving rise to the contractions that pump blood to the body. Spiral waves, which are self sustained reentrant pulses that rotate freely in the medium, are special abnormal propagating waves and are believed to be the source of dangerous arrhythmias [6], [7]. A particular problem arises when spiral waves become unstable giving rise to chaotic patterns that are associated with fibrillation [8].

These waves often interact with non excitable objects that are usually referred to as obstacles [9]. The interaction of spiral waves with obstacles in cardiac dynamics is a topic that has been deeply investigated in the recent years. Its importance relies on the fact that the presence of obstacles might alter the inherent dynamics of the spiral wave. For example, spiral waves might develop when periodic stimulation takes place in a medium with obstacles [10, 11]. Also, it has been observed that an obstacle in cardiac tissue might act as a stabilizer of spiral wave dynamics [12, 13, 14, 15, 16], as it provides a transition between meandering spiral waves [12] or multiple spiral waves [16, 17] into a simple rotation spiral, which is attached to the obstacle. This transition is clinically important because as it has been shown, fibrillation like activity changes to a tachycardia regime [18]. In the same way, the presence of small random obstacles might help to prevent the phenomenon of spiral break up [11]. Recent computational studies [9, 10] have shown that spiral waves might act also as destabilizers of spiral waves. Such instabilities might be as simple as having a transition of a one to two frequencies rotation and transitions from two to three frequencies, or transitions from two frequencies to a completely irregular pattern [9]. In these cases the transitions arise as a local interaction of the tip of the spiral wave with the obstacle and the instability is completely characterized by the tip of the spiral wave.

In this work, we present a simple model based on the Fitzhugh-Nagumo (FHN) type local dynamics to show three different behaviors when a spiral wave in a particular regime interacts with an obstacle: (i) the stabilization of the spiral wave dynamics due to attachment, (ii) the bouncing of the spiral at the obstacle and (iii) spiral break up due to the interaction. From these properties, the first two arise from local interaction, whereas the last one is a non local phenomenon. These results not only provide with evidence that obstacles might act as stabilizers or destabilizers of the spiral wave dynamics, but also in the case of destabilization, new vortices might arise, giving a less controllable scenario.

Therefore, this work is organized as follows. We initially consider the model equations and the numerical scheme developed for the simulations. Then, a discussion of the inherent dynamics of the spiral wave given by two of the parameters involved is presented. After that, the results of considering interactions of spiral waves with obstacles are presented and discussed. We end this work with a discussion and conclusions section.

2 Model Equations and Numerical Methods

The FHN equations are the simplest model to describe the excitation in a cell due to the flux of sodium and potassium ions through the ionic channels [19]. This model, which initially was proposed for nerve cells is used to model excitable media in general. Despite the simplicity of the FHN equations, they keep being used to show different generic behavior in excitable media. Experience has shown that the results obtained with generic equations such as the FHN, are preserved for more realistic models [10]. Equations of the FHN type have been considered theoretically and computationally to study the interaction of spiral waves with obstacles. Panfilov and Keener studied the generation of spiral waves due to periodic stimulation of an obstacle [10]. Feng et al [7] used equations of the FHN type and the LR equations to model the unpinning and elimination of spiral waves attached to obstacles. Gao and Zhang [20] used the FHN equations to propose a theoretical explanation of unpinning of spiral waves to obstacles. Das et. al. [21] studied the unpinning of scroll waves under the influence of a thermal gradient. Pumir and Krinsky [22] studied the unpinning of a rotating wave in cardiac muscle by an electric field. Barkley [23] present a detailed analysis of spiral waves in excitable media, and propose a low-order system of differential equations to describe spiral dynamics. This model is widely used to study the electrical activities of cardiac tissues [24].

The model in this work is a modification of the proposed by Barkley [25], denoted as MFHN, and is given by

$$\begin{aligned} \frac{\partial u}{\partial t} &= D\nabla^2 u + \frac{1}{\epsilon}u(1-u)(u-u_{th}) \\ \frac{\partial v}{\partial t} &= \delta(u,v)(u-v) \end{aligned} \tag{2.1}$$

where $u_{th} = \dfrac{v+b}{a}$, $a, b \neq 0$ are dimensionless quantities and $\delta(u,v)$ controls the recovery time of variable v and is given by

$$\delta(u,v) = \begin{cases} \delta_1 = 0.3 & \text{if } u < 0.2 \text{ and } v > 2 \\ \delta_2 = 5 & \text{if } u < 0.2 \text{ and } 0.05 \leq v \leq 0.2 \\ \delta_3 = 1 & \text{other case} \end{cases}$$

In the case of $\delta(u,v) = 1$, the proposed model reduces to the original Barkley equations [25]. Modifications of the Fitzhugh-Nagumo models have been considered. Panfilov and Keener [10], used a modified version to study spiral break up due to the high frequency stimulation of an inexcitable obstacle; and Bär and Brusch [26] analyzed different mechanisms for the breakup of spiral waves.

The set of equations 2.1 was considered with a given initial condition and at the boundary, no flux boundary conditions are taken. These equations were solved numerically by finite differences in the domain $\Omega = [-10, 10] \times [-10, 10]$ where 512 discretization points were taken in each direction. The time integrator was the Euler method with time step $\Delta t = 0.0001$. Rectangular obstacles inside Ω were implemented with no flux boundary conditions.

In all the experiments where obstacles are considered, the same initial condition was taken to generate the spiral wave. This was done by taking the initial condition

$$\begin{aligned} U(x,y) &= \left(\tfrac{1}{1+exp(4|x+\alpha y|-r_1)}\right)^2 - \left(\tfrac{1}{1+exp(4|x+\alpha y|-r_2)}\right)^2 \\ V(x,y) &= 0 \end{aligned} \tag{2.2}$$

where $r_1 = 9.8, r_2 = 9, \alpha = 0.414214$ and the variable $U(x, y)$ is redefined as zero for $\alpha y > -x$. The resulting front evolves and at time $t = 1$ for $\Delta t = 0.1$ time units, the solution is redefined as $U(x, y) = 0$ and $V(x, y) = 0.25$ for $\alpha x < y - 5$. By doing this, a spiral wave is obtained for all the experiments in this work.

3 Spiral Tip Dynamics

By modifying the values of a and b it is possible to do an analysis of the different trajectories of the tip of the spiral. A very important characteristic of the spiral wave is the trajectory of its tip, which describes how wave propagation follows. These tip trajectories can be located on an approximation given by the intersection of two level curves $u = 0.5$ and $v = 1$ [25]. The analysis of the tip trajectory of the non modified version has been studied in [25]. If we change the values of a between 0.5 and 0.9, with $b = 0.05$ and $b = 0.1$, different trajectories are formed. The general behavior of the tip trajectories for different values of a and b are summarized in Fig. 1.

In the case where $b = 0.05$ and $a \in (0.5, 0.54)$, the tip of the wave trace circles of radius r^*, which decreases as a increases. When $a > 0.57$, the circle degenerates and draws an epitrochoid with radius R. When a increases, R begins to decrease until the tip of wave forms a trochoid. If a continues to increase and reaches a value close to 0.66, the tip forms a hypotrochoid of radius R, with many petals. Since R is very large compared with the length of r, the parameter a continues to increase and the radius R decreases. When this happens, the number of petals decreases down to three, with $a \approx$0.74. After this value, there is a degenerate form of the hypotrochoid of radius R, and so continued until a takes the value of 0.9.

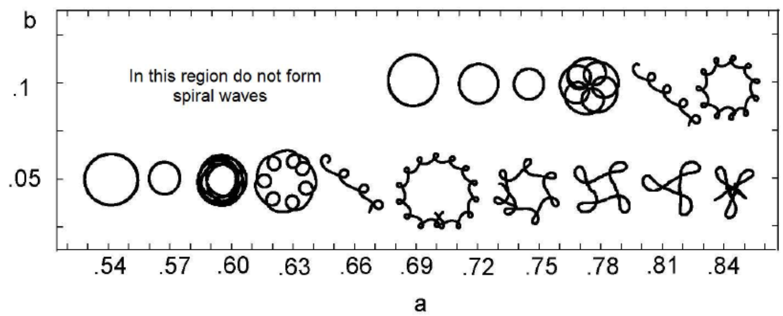

Fig. 1. Dynamic modeling of the spiral tip with MFHN equations for $a \in$ (0.5, 0.9) and $b =$ 0.05 (lower row), $b =$ 0.1 (upper row)

The propagation of an action potential has an initial phase of excitation. With the passage of potential, the excited region enters a refractory state, during which it may not be elicited another action potential. Finally, the region enters the recovery phase. At this stage, propagation of action potentials may or may not occur, depending on the recovery level of the medium.

The consequence on the spiral wave propagation due to different recovery levels of the medium is shown in Fig. 2 for a spiral wave in the trochoidal regime ($a = 0.829, b = 0.1$ and $\epsilon = 0.2$). The trochoidal regime can be seen as an inherent drift of the spiral wave and will be helpful to understand the interactions between spiral waves in the meandering regime and non-excitable obstacles. The black curve is the trajectory traced by the tip of the spiral during the simulation time. The location of the tip of the spiral lies on the level curve $u = 0.5$ (yellow curve) and is shown with the black dot filled in white. The level curve $u = 0.1$ (blue curve) gives the location of the bottom part of the wavefront, which is used to know if a front is able to propagate in a determinate region. The back of the wave becomes distinguishable from the wavefront at the location close to where the level curve $u = 0.5$ hits the tip of the spiral wave. The different gray scaled regions represent different levels of refractoriness of the medium given by the v variable, where the black region means the highest level of refractoriness $v > 0.5$ and white which means a completely recovered region $v \in [0, 0.03)$.

The region in dark gray ($v \in (0.4, 0.5)$) indicates that the medium is recovering but is not ready for a new action potential. The medium gray region with $v \in (0.1, 0.4)$ indicates that the medium is fairly recovered. An AP is difficult to be elicited in this region. The region with $v \in (0.03, 0.1)$ (light gray) indicates that the medium is almost recovered and therefore, a new action potential can spread. Finally, the white region represents fully recovered medium so in this region another action potential can easily occur.

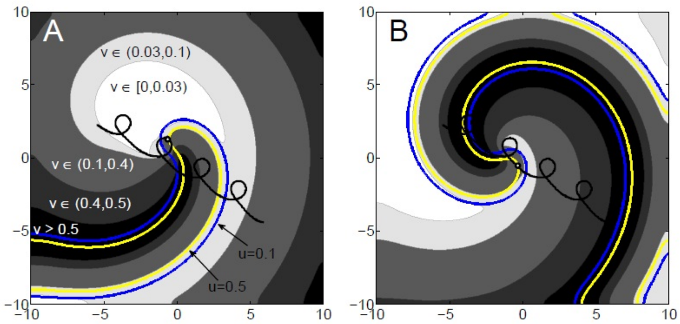

Fig. 2. Propagation of a spiral wave by using MFHN equations with parameters a=0.83, b=0.1 and ϵ= 0.02. The tip of the spiral wave is located at the black dot filled in white. The different gray scaled colors represent different levels of refractoriness given by the v variable; the black region indicates the highest level of refractoriness ($v > 0.5$), whereas the region in white is the most recovered medium ($v \in [0, 0.03)$). A) Conformation of a petal. B) Conformation of an arc

In Fig. 2A we can see that the tip of the spiral trace a high curvature loop trajectory, known as a petal. In this case, the wave front near the tip propagates in a region completely recovered causing the formation of the petal. In Fig. 2B the trajectory of the tip traces a curve with less curvature, known as an arc, than in Fig. 2A. In this case an arc is formed because the tip of the spiral reaches the back of the wave, which is a region not totally recovered. This causes the spiral fails to adequately spread and begins to move looking for an excitable region. After a while, the neighborhood around the tip of the spiral recovers completely, causing a phenomenon similar to that seen in the Fig. 2A and the process repeats. An interesting phenomenon that occurs during the formation of a petal, is that the front near the tip of the spiral propagates over an almost recovered medium, contrary to the front far from the tip, which propagates over a non-recovered medium. Also, when the tip trajectory generates an arc, the front near the tip propagates over a nonrecovered medium, whereas the front far from the tip propagates over an almost recovered medium (Fig. 2).

4 Interaction of Spiral Waves with Obstacles

Once it was presented the behavior of the spatial dynamics of the model, we proceed to present numerical studies of the interaction of a spiral wave in the trochoidal regime with a non-excitable obstacle. The study of such interaction has been considered previously in [14], for equation 2.1. In that study it was only considered the case where the interaction takes place when the tip traces a petal. However, the case when the interaction occurs with the tip tracing an arc has not been considered. In the present case, there are different and more complex behaviors than in the cases studied in [14]. It is shown that not only attachment or bouncing is observed but also new spirals are born.

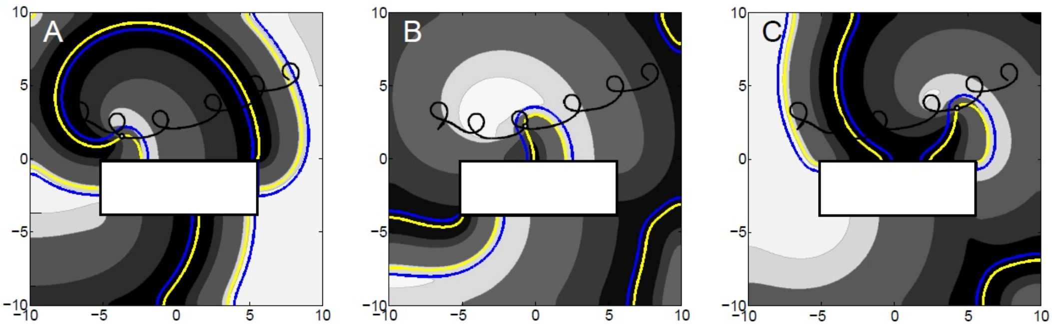

Fig. 3. Bouncing of a spiral wave at an obstacle. A local approximation of a hypotrochoidal trajectory with a trochoidal trajectory. Dynamics obtained with the MFHN equations and parameters a= 0.83, b = 0.1, ϵ= 0.02

4.1 Bouncing or attachment of a spiral wave in presence of an obstacle

By using the MFHN equations with parameters $a = 0.83, b = 0.1$ and $\epsilon = 0.02$ we simulated the interaction of a spiral wave with an obstacle when the interaction takes place when the spiral wave is tracing an arc. The obstacle has coordinates $\Omega_o = [-5.1467, X_R] \times [-3.8551, -0.1369]$. In Figs 3 and 4 we present two scenarios.

In both cases it is considered the same initial condition to generate the spiral wave which will interact with obstacles of different size. The obstacle in Fig. 4 is much shorter than the one shown in Fig. 3. The coordinates X_R are 5.6164 and -2.9941 for fig. 3 and fig 4, respectively. The reason of this choice is to maintain the same dynamics of the core of the spiral wave until it interacts for the last time with the obstacle, and then conclude the differences given solely by the site of interaction.

The first frame (Fig. 3A) shows the spiral wave after the first two rotations, where the beginning of the interaction of the tip of the spiral and the obstacle takes place. Here it is noted that the tip is tracing an arc. At this time, the presence of the obstacle generates a gain in the curvature of the tip trajectory as studied by Yermakova and Pertsov [27]. Due to this obstacle effect, the trajectory of the tip changes its inherent drift direction leading the spiral to move away from the obstacle. In Fig. 3B and 3C, are shown two frames after the change of direction has happened. From the figures it is clear that there are no more effects on the tip trajectory due to the presence of the obstacle, and therefore, the spiral has bounced off the obstacle.

The second scenario is shown in Fig. 4. In Fig. 4A it is shown the solution before the second petal is traced. Observe that the front that is close to the tip of the spiral, propagates close to a corner. The spiral propagates generating the second petal. In fig. 4B, the spiral has just generated a petal and in this case, the front close to the tip trajectory has a shorter length than in the previous frame.

Added to this, the tip will try to generate an arc as the region where the front propagates is now less recovered and the area to excite is larger as we are at a corner. Due to these issues, the front cannot propagate properly and the front detaches to the obstacle reducing its strength (Fig. 4C). Finally, the front dies out, and the only remaining spiral wave is the part that its attached to the obstacle.

From these two simulations it is important to observe that the size of the obstacle is not the factor to obtain bouncing or attachment of the spiral wave to the obstacle. What really matters is the fact that the geometry at the site of interaction plays a crucial role in the process of attachment and bouncing. This conclusion has been noted already in [14], where the interaction took place with a petal rather than an arc.

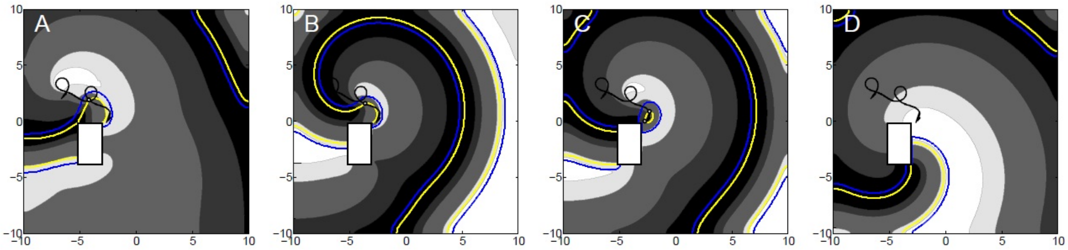

Fig. 4. Attachment of a spiral wave to an obstacle. A local approximation of a hypotrochoidal trajectory with a trochoidal trajectory. Dynamics obtained with the MFHN equations and parameters a= 0.83, b = 0.1, ϵ= 0.02

4.2 Generation of two vortices

In Fig. 5 we show what happens when we take an intermediate size obstacle in the horizontal direction. The obstacle is now given by $\Omega_o = [-5.1467, X_R]\times[-3.8551, -0.1369]$, with $XR = 1.6242$. We focus particularly on the interaction near a corner of the obstacle. In this case, we show initially the spiral wave after it has changed its tip trajectory direction due to the boundary effects that the obstacle imposed to the trajectory (Figure 5A). As the front moves towards the corner, it is observed that a larger area needs to be excited (Fig. 5B). In this case, two factors enter the game. i) the medium is not totally recovered which gives detachment, and ii) the length of the front is large enough which prevents that the front does not die out as in Fig. 4. The final result is shown in Figs 5C and 5D, where there are two free ends, which evolve in a pair of spirals.

Therefore, by increasing the length of the horizontal size of the obstacle, we have obtained three different behaviors. Bouncing of the spiral wave, attachment of the spiral wave to the obstacle and generation of new vortices. Even that the three regimes are of interest, there is the question of the nature of the results. Are the results merely a product of local interaction? In the case of attachment and bouncing, the answer is affirmative. Attachment is due to the facts that the tip has just traced a petal and the region next to the propagating front is not totally recovered, and the interaction happens at a corner of the obstacle. Also, attachment might occur if there is no corner involved but the angle of incidence and the phase tip trajectory are the appropriate [28, 14]. Bouncing follows from the fact that corners are not present and the angle of incidence and the phase of the spiral wave are appropriate [28, 14]. However, the generation of a new spiral is not completely a local effect. Even that in Fig. 5, local arguments were used, the scenario can be seen in a more global scale.

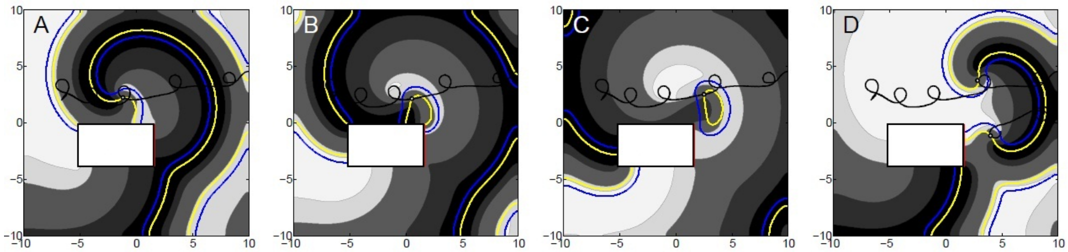

Fig. 5. Generation of a new spiral wave after interaction with an obstacle. The values of a, b, ϵ the same as in the previous figures

4.3 Generation of two vortices as a non local effect

In the previous section it was shown that two vortices were born when the meandering spiral interacted with the obstacle. However, in this section we discuss the phenomenon by taking into account the size and shape of the obstacle. To this end, and additional to the phenomenon in Fig. 5, it is shown in Fig. 6 the generation of an extra spiral wave except for the fact that the new spiral is born at the lower right corner of the obstacle.

The main idea behind the generation of the new spiral, lies on the fact that the area at the lower corner (Fig. 6B) is not completely recovered from the previous front, where the local argument presented in the previous section follows. However, how do we see the nonlocal situation? Figs 6 and 7 are used provide an answer to such question. The obstacles in figs 6 and 7 have coordinates $\Omega_o = [-5.1467, 3.9726] \times [Y_L, -0.1369]$ with $Y_L = -3.1898$ and -6.5166, respectively, i.e. the obstacle in Fig. 7 is larger in the vertical direction than the one shown in Fig. 6.

The consequence of the different vertical lengths is that for the shorter obstacle detachment and generation of a new spiral is obtained, whereas for the larger obstacle, there is no detachment. In both cases, the dynamics of the tip of the spiral is the same. However, when the fronts that move down (on the right side of the obstacle), meet the corner (Figs. 6B and 7A) they find the region to excite with different levels of refractoriness, giving the presented results. In the case of fig. 7A, the front has to travel more in order to arrive to the corner, and therefore, there is more time for the medium at the corner to recover from the previous front.

From the discussion in the previous paragraph it is clear that the vertical length of the obstacle plays an important role in the generation of a new spiral wave. Detachment is observed for $l < l^*$, whereas for $l > l^*$ there is no generation of new spirals. The horizontal length plays an important role as well and this is seen by discussing the events in Fig. 6. Fig. 6A, shows the spiral wave just at the end of the third petal. In this interaction there are two attached fronts that propagates to the right and to the bottom, respectively (Fig. 6A, arrows). At the same time, the previous two fronts just collided at the lower right corner. These last two collided fronts came from a similar situation than the two attached fronts in the figure but from a previous spiral rotation, with the difference that they were generated closer to the upper left corner than the actual two presented in fig. 6A. In fig. 6B it is shown the spiral just before the fifth petal is being formed. As the core of the spiral wave has moved to the right, now the two attached fronts were generated closer to the upper right corner. Moreover, the front moving to the right has reached the previous front (Just as in the case in Fig. 2B). Therefore, the front reaches the lower right corner of the obstacle, where the medium is not totally recovered (Fig. C), obtaining detachment of the front at the corner.

The key point is that now the attached front on the left, is quite far from the lower right corner, due to the distance that had to run, and therefore when detachment occurs, the fronts do not annihilate among them. To end the discussion, in fig. 6D it is shown two meandering spiral waves due to the interaction of one spiral with the obstacle. Therefore, we conclude that the generation of a new spiral in the medium is not a local effect, but a consequence of the size and shape of the obstacle.

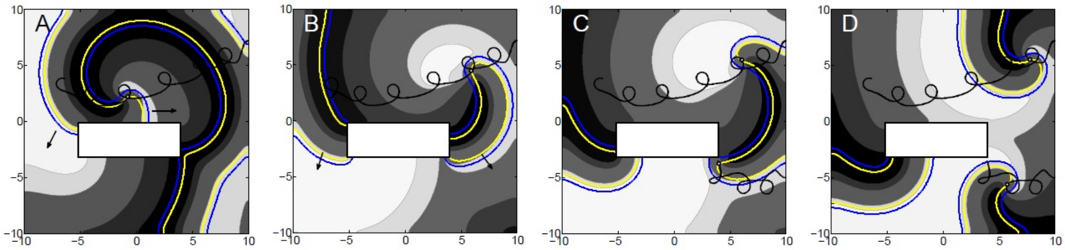

Fig. 6. Breaking the spiral wave is after having interacted with an obstacle without being attached. Generation on the two spirals rotating freely

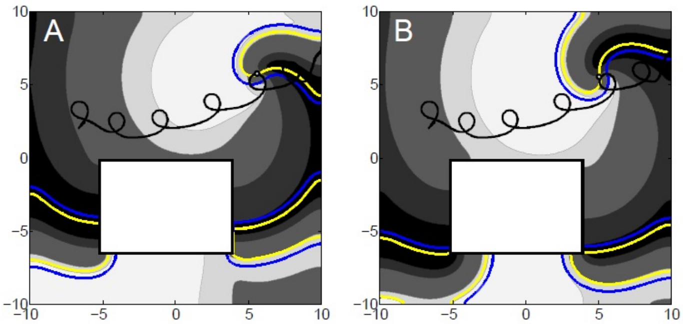

Fig. 7. Breaking the spiral wave is after having interacted with an obstacle without being attached. Generation on the two spirals rotating freely

5 Discussion

In this paper it was studied possible scenarios when a meandering spiral wave, tracing a trochoid, interacts with rectangular obstacles. This problem has been studied in [14], but the interactions only considers that the spiral wave hits the obstacle when a petal is being traced. In this document, we show that the interaction can also take place when the tip is tracing an arc. The results though similar to those in [14], can produce other results which have not been reported in the literature. In this work, it was modified the length of an obstacle and from there, three different behaviors were obtained. Attachment to the obstacle, bouncing, and generation of new vortices in the medium. Cysyk [29] have proposed an experimental method to avoid attachment of spiral waves.

The interaction of spiral waves and obstacles is a difficult task to understand completely. For simplicity, in this work it was considered a tip trajectory in the trochoidal regime. This was done in order to approximate the behavior of the hypo and epitrochoidal regimes in the local interaction with a boundary.

For the case of attachment and bouncing, there are a couple of notes to consider. From previous work [14, 28], it follows that when a spiral wave in the trochoidal regime hits a boundary (obstacle far from a corner), it does it with an incident angle and might bounce or attach to the obstacle. Therefore, the effect of bouncing does not depend only on the site of interaction (far from a corner), but also depends on the angle of incidence and phase of the spiral wave [28]. In this work it was considered only one incident angle, and one phase giving a particular bouncing angle. For the purposes of the work presented here, it is enough to present this particular example to show the different scenarios that might occur.

Attachment was shown in the case when the tip of the spiral wave hits near to a corner or at a corner. However, attachment might occur as well if the interaction takes place far from a corner, depending of the angle of incidence and the phase of the spiral wave. Also, interaction of the tip of a spiral wave with an obstacle at a corner, will in general give attachment, but this is not a rule. There might be examples where bouncing is possible.

The third scenario obtained is the emerging of a new spiral wave additional to the one remaining in the medium. The generation of the extra vortex gives a less controllable arrhytmic scenario. The mechanism by which this happens has a nonlocal nature. Therefore, in order to deep our understanding in the interaction spiral wave - obstacle, and to increase our knowledge about mechanisms of spiral wave break up, it is necessary to explore the nature of the obstacles present in the medium. Features like the regime of the spiral wave, the size, the shape and the place where the interaction takes place, might play an important role in the generation of new vortices. Hornung et al [30] proposed a solution to multiple vortices in the heart.

In the case of epitrochiodal and hypotrochoidal regimes the interaction of spiral waves with obstacles is more complex as repetitious interactions may take place as shown in [9]. The trochoidal regime is useful only for local interactions of the other two regimes, and as the spiral breakup is a nonlocal effect, the trochoidal regime cannot approximate the epitrochoidal and hypotrochoidal regimes. Moreover, the effect of spiral break up was not observed for the hypotrochoidal and epitrochoidal regimes. It seems then that the phenomenon of spiral breakup is useful only for dynamics in the trochoidal or close to the trochoidal regimes. This restriction appears to reduce considerably the applicability of the findings in this paper even for more complex models for excitable media.

Despite the restriction discussed in the last paragraph, it is possible to find useful the obtained results, particularly for the case where drift of a spiral wave takes place. Drift of spiral waves has been observed experimentally in cardiac tissue [31] and also in the Belousov-Zhabotinski reaction [32]. By using mathematical models, meandering and drifting spiral waves can be obtained by time periodic coupling strength [33] and due to the presence of a rotating electric field [34]. Calvo et. al. [35] studied mechanisms of rotor drift due to spatial gradients in the currents, whereas drift of a spiral wave due to periodic stimulation was studied by Gottwald [36]. Additionally to this list, one can refer to the work by Biktashev [37] for more information about drift of spiral waves.

Under this scenario, it is possible to state the question whether spiral break up occur or not when a drifting spiral wave moves close to an obstacle. Certainly, the mechanisms of drift are not the same than the one that generates a meandering spiral wave in the trochoidal regime, but the break up phenomenon presented in this work takes into account factors that were merely related to the size and shape of the obstacle and drift of a spiral wave. Therefore, the value of this work is to know that drifting spirals might experience breakup when their core moves close to an obstacle.

An important point to mention is that the phenomenon of spiral break up studied in this work can be compared with the experiment of periodic stimulation as the work in Panfilov and Keener [10]. However, the mechanism presented here and in [10] are different as the source of the periodic stimulation in this work is not fixed as in [10] but moves in time.

Finally, based on the propagation properties of the excitable media and a size and shape of present obstacles, it should be possible to argument whether the spiral break up scenario is likely or not to happen. Lim et al [13] studied spiral waves attached to millimeter-sized obstacles and its relationship with obstacle-size.

The results obtained in this work helps to understand better the interaction between spiral waves and non excitable obstacles. Particularly, can be helpful to prevent spiral break up when drift is present and there are obstacles in the medium.

6 Conclusion

A spiral wave, which is a self-sustaining wave, generates certain types of arrhythmias, which can lead to ventricular fibrillation. Spiral waves are also related to atrial flutter and atrial fibrillation. It is known that arrhythmias caused by spiral waves can cause fibrillation, which is a completely disorganized stimulation of cardiac tissue. In this paper, we study a generic model for the propagation of electrical impulses in cardiac tissue based on the Fitzhugh-Nagumo equations. By numerical studies we consider the problem of the generation and propagation of spiral waves. Finally, we analyze the case of having the presence of obstacles, such as ischemic or dead tissue from a heart attack or surgery. We present an equation which is a modified version of the Fitzhugh-Nagumo equation, varying a parameter that controls repolarization time. With these results it is possible to give a physiological interpretation of the parameters of the Fitzhugh-Nagumo equation. Likewise, there is a spiral wave interaction with inexcitable obstacles. Finally, we propose two new mechanisms for destabilization of a spiral wave, due to the presence of obstacles.

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Competing Interests

Authors have declared that no competing interests exist.

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