



## Velocity Profiles of Unsteady Blood Flow through an Inclined Circular Tube with Magnetic Field

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### Authors' contributions

*This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.*

### Article Information

DOI: 10.9734/JAMCS/2017/36620

#### Editor(s):

(1) M. Subhas Abel, Department of Mathematics, Gulbarga University, India.

#### Reviewers:

(1) Ajaz Ahmad Dar, Annamalai University, India.

(2) Jagdish Prakash, University of Botswana, Botswana.

Complete Peer review History: <http://www.sciedomains.org/review-history/21302>

**Received: 4<sup>th</sup> September 2017**

**Accepted: 27<sup>th</sup> September 2017**

**Published: 10<sup>th</sup> October 2017**

**Original Research Article**

## Abstract

The present paper is devoted to study the flow of incompressible viscous, electrically conducting fluid (blood) in a rigid inclined circular tube with magnetic field. The blood is considered to be Newtonian fluid and the flow is caused by varying pressure gradient with time. The physics of the problem is described using the usual Magneto hydrodynamic (MHD) principles and equations along with appropriate boundary conditions. The governing equations of the motion in terms of cartesian co-ordinates are reduced to ordinary differential equations. Using dimensionless parameters, the Navier-stokes equation is solved numerically using finite difference method of approximation and the expressions for velocity profile is obtained. The velocity profiles for various values of Hartmann number as well as varying the angle of inclination of the tube have been presented graphically and discussed in depth. The obtained results show that on increasing the inclination angle of the tube and Hartmann number leads to increase and decrease of the axial velocity of the blood respectively.

*Keywords: Biomagnetic fluid; incompressible viscous fluid; Newtonian fluid; Hartmann number; inclined magnetic field.*

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## Nomenclatures

$B_o$	External magnetic field strength, [ $Wbm^{-2}$ ]
$\vec{B}$	Vector of magnetic induction, [ $Wbm^{-2}$ ]
$\frac{\partial \vec{B}}{\partial t}$	Rate of change of magnetic flux, [ $Wb$ ]
$C$	Velocity of light, [ $ms^{-1}$ ]
$E$	Electric field strength, [ $Vm^{-1}$ ]
$\vec{E}$	Electric field strength vector, [ $Vm^{-1}$ ]
$\vec{F}$	Electromagnetic force, [ $N$ ]
$Fr$	Froude number, [ $= \frac{w_o^2}{gL}$ ]
$\vec{F}_B$	Body force per unit mass of fluid, [ $N$ ]
$g$	Gravitational acceleration, [ $ms^{-2}$ ]
$Ha$	Hartmann number, [ $= LB_o \sqrt{\frac{\sigma}{\mu}}$ ]
$\vec{j}$	Current Charge density, [ $Am^{-2}$ ]
$L$	Length of the tube, [ $m$ ]
$P$	Pressure of blood flow, [ $Nm^{-2}$ ]
$P^*$	Dimensionless pressure
$x_o$	Radius of the tube, [ $m$ ]
$x^*$	Dimensionless radius
$Re$	Reynolds number, [ $= \frac{Lw_o\rho}{\mu}$ ]
$t$	Time, [ $s$ ]
$t^*$	Dimensionless time
$w$	Axial velocity of blood flow, [ $ms^{-1}$ ]
$w^*$	Dimensionless velocity
$w_o$	Representative Value of velocity in the tube, [ $ms^{-1}$ ]
$\vec{W}$	Vector velocity of blood flow, [ $ms^{-1}$ ]
$\frac{\partial w}{\partial t}$	Unsteady acceleration of blood flow, [ $ms^{-2}$ ]
$x, y, z$	Cartesian co-ordinates
$Z$	Direction of blood flow
$Z^*$	Dimensionless direction of blood flow
$\alpha$	Angle of inclination of magnetic field, [degrees]
$\Delta t$	Time interval, [ $s$ ]
$\Delta x$	Radius interval, [ $m$ ]
$\nabla$	Differential operator
$\nabla P$	Pressure gradient, [ $\nabla P = \frac{\partial p}{\partial z}$ ]
$\nabla^2$	Laplacian operator
$\epsilon_0$	Electric permittivity of free space, [ $Fm^{-1}$ ]
$\theta$	Angle of inclination of circular rigid tube, [degrees]
$\mu$	Co-efficient of viscosity of blood, [ $kgm^{-1}s^{-1}$ ]
$\mu_0$	Magnetic permittivity of free space, [ $Hm^{-1}$ ]
$\mu_e$	Magnetic permeability of the fluid, [ $Hm^{-1}$ ]
$\rho$	Density of blood, [ $kgm^{-3}$ ]
$\rho_e$	Charge density, [ $Cm^{-1}$ ]
$\sigma$	Electrical conductivity of the blood, [ $\Omega^{-1}m^{-1}$ ]

## I. Introduction

Biomagnetic Fluid Dynamics (BFD) is an area in fluid mechanics that investigates the fluid dynamics of biological fluids in the presence of magnetic field while Electromagnetism on the other hand studies the interaction between the electric and magnetic fields. A bio-magnetic fluid (BMF) is a fluid that exists in living creatures and its flow is influenced by magnetic field. Blood is considered to be a bio-magnetic fluid because the red blood cells contain the haemoglobin molecules, a form of iron oxides which are present at high concentrations in the mature red blood cells. The study of MHD flow problems through tube has found an application in many fields like MHD power generation, blood flow measurements etc.

The idea of the effect of electromagnetic fields on blood was firstly raised by Kolin [1], who studied electromagnetic flow meter and its applications to blood flow measurement. He established that biological systems are greatly affected by external magnetic field and increasing the strength of magnetic field was found to decrease the blood flow velocity. Later, Korchevskii and Marochnik [2], discussed the possibility of regulating the movement of blood in human circulatory system by applying magnetic field and concluded that the flow of blood in human circulatory system can be regulated by applying appropriate magnetic field. Vardanyan [3], investigated the effect of magnetic field on the blood flow theoretically and later collaborated with Sud et al. [4], to develop different models. Suri [5], examined effects of static magnetic field on blood flow in a branch. He demonstrated that increasing the magnitude of Hartmann number reduces the strength of blockage at the apex of bifurcation, shear stress and speed of blood.

Sanyal et al. [6], studied effect of transverse magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration and in which they considered blood as coupled stress fluid. They showed that the flow velocity of blood deviates with various parameters such as Hartmann number, gravitational parameter, body acceleration, inclination angle and time. They concluded that the deviations can be regulated by use of appropriate magnetic field. Dar and Elangovan [7], investigated the influence of an inclined magnetic field and rotation on the peristaltic flow of a micropolar fluid in an inclined channel. The study mainly concerned with the interaction of both rotation and inclined magnetic field on peristaltic flow of a micropolar fluid in an inclined symmetric channel with sinusoidal waves moving down its walls. They adopted low Reynolds number and long wavelength in simplifying in order to attain nonlinear equations. The analytical and numerical solutions for axial velocity, spin velocity, volume flow rate, pressure gradient, pressure rise per wavelength, and stream function were computed and analyzed. The quantitative effects of various embedded physical parameters were inspected and displayed graphically with fussy prominence. They concluded that pressure rise, frictional forces, and pumping phenomenon increased as both inclination angle increases respectively.

Various mathematical models have been investigated by several researchers to examine the behavior of blood flow under influence of magnetic fields (Bali and Awasthi [8], Tanwar et al. [9], Tripathi [10], Shit [11], Sharma et al. [12] Srivastava [13]). Kumar et al. [14], studied mathematical modeling of blood flow in an inclined tapered artery under MHD effect through porous medium. They considered blood as an electrically conducting Newtonian fluid and used analytical expressions to derive volumetric flow rate, pressure gradient, wall shearing stress, and velocity profile. They concluded that an effect of permeability on the volumetric flow rate, axial velocity and flow patterns in non-tapered region, converging region, and diverging region were effectively influenced by the presence of magnetic field and change in leaning of artery. The research was found helpful for the practitioners to treat the hypertension patient through magnetic therapy and to understand the flow of blood under stenotic conditions.

Wahab and Salem [15], investigated steady MHD blood flow through a narrow tube and considered blood as incompressible, viscous and electrically conducting fluid in presence of transverse magnetic field. They used cylindrical polar coordinates to define governing equations and reduced it to ordinary differential equations by using dimensionless parameter, Hartmann number and then used analytical method to solve the governing equations. They found out that increasing magnetic field led to decrease in velocity and flow rate of blood. Jyothi et al. [16], discussed the effect of magnetic field on the peristaltic flow of a Newtonian fluid in an

inclined tube. They noted that the time average flux volume flow rate increased with Hartmann number, angle of inclination and Reynolds number while it decreased with increase in Froude number. The study contributed much to knowledge of MHD in medical and a bio-engineering science which is associated with the development of magnetic devices, hyperthermia, cancer tumor treatment and blood reduction during surgeries. Dar and Elangovan [17], investigated the influence of an inclined magnetic field on heat and mass transfer of the peristaltic flow of a couple stress fluid in an inclined channel. In their methodology they used long wavelength and low Reynolds number for simplifying the highly nonlinear equations. They derived mathematical expressions of axial velocity, pressure gradient and volume flow rate. Pressure rise, frictional force and pumping phenomenon were portrayed and determined together with exact and numerical solutions. The computed results were presented graphically for various embedded parameters together with temperature and concentration profile. The study concluded that the fluid motion can be enhanced by increasing the inclination of both the magnetic field and the channel. Dar and Elangovan [18], studied the impact of an inclined magnetic field, heat generation/absorption and radiation on the peristaltic flow of a Micropolar fluid through a porous non-uniform channel with slip velocity. They considered incompressible peristaltic flow of a micro-polar fluid through a permeable non-uniform channel in the vicinity of an inclined magnetic field with heat and mass transfer. The effects of heat generation, radiation and spin velocity on the fluid with presumptions of long wavelength and low Reynolds number approximations were studied. Mathematical expressions for axial velocity, micro-rotation speed, pressure gradient, volumetric stream rate, temperature and concentration were described. The impact of different applicable physical parameters was dissected hypothetically and figured numerically then results were plotted graphically. The outcome determined that the impact of Magnetic field, coupling number, micropolar parameter, slip parameter, inclination of magnetic field parameter, porosity parameter, heat generation and thermal radiation parameter was extremely protuberant in the study.

Verma and Srivastava [19], investigated effect of magnetic field on steady blood flow through an inclined circular tube. They considered blood to be Newtonian, incompressible, viscous and electrically conducting fluid flowing under the presence of transverse magnetic field. The governing equations were solved analytically to obtain velocity profile and flow rate. They found that an increase of Hartmann number and inclination angle of a circular tube led to bluntness of velocity profile. They pointed out that applying appropriate magnetic field to blood transfusion instruments can regulate blood flow and enhance a patient's recovery. Extensive researches have been done, including those cited above. However, no emphasis has been given to unsteady blood flow and keeping in view of medical importance in the application of MHD principles as mentioned above. Then, the objective of this research is to study the velocity profile of blood flow through an inclined circular tube with magnetic field.

## 2. Mathematical Formulation

Let's consider unsteady, one dimensional fully developed flow of an incompressible viscous blood through a uniform straight and inclined rigid circular tube in the presence of an external uniform inclined magnetic field. The flow is axially symmetric and the blood is modeled to be a Newtonian fluid in a non-conducting tube. The governing equations are derived using Cartesian co-ordinates  $x, y, z$  where  $z$  lies along the centre of the tube. The tube is filled with blood of electrical conductivity  $\sigma$ , density  $\rho$ , coefficient of viscosity  $\mu$ , and moving with axial velocity  $w(x, t)$ . The diameter of the tube is assumed to be greater than  $1mm$  so that the Fahraeus-Lindqvist effect is not significant and the temperature of blood is assumed to be uniform. The blood is assumed to flow in  $z$  – direction as shown in figure A below.

The blood is acted upon by pressure gradient  $\partial p / \partial z$  which varies with time and is injected through the tube at  $t = 0$ . A uniform magnetic field  $B_0$  is applied in the direction making an acute angle  $\alpha$  with the  $z$ -axis.

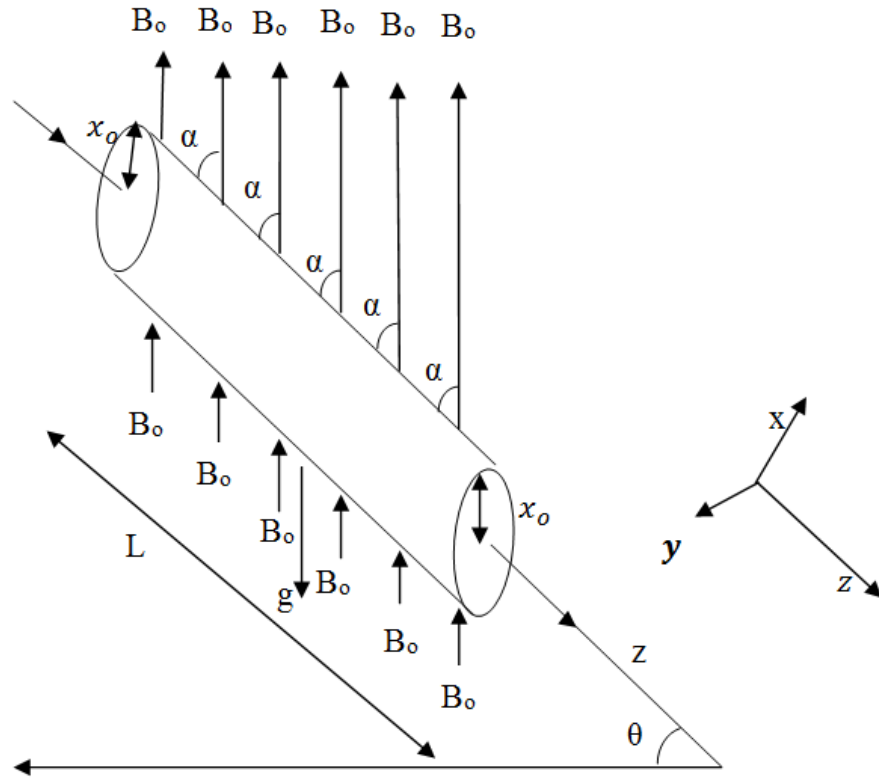


Fig. A. Schematic model for the flow geometry

### Continuity equation

The continuity equation is written as:

$$\frac{\partial \bar{w}}{\partial z} = 0 \quad (1)$$

### Momentum equation

The momentum equation in one-dimension flow reduces to:

$$\rho \frac{Du_z}{Dt} = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial x^2} + \rho g_z \quad (2)$$

The magnetic Reynolds number is assumed to be negligible so that the induced magnetic field is neglected. It is also assumed that no electric field exists and the Hall effect is neglected. The uniform magnetic field  $B_0$  is inclined to the tube at an angle  $\alpha$ . For electrically conducting blood in the presence of magnetic field, the interaction between the current density  $\vec{j}$  and the magnetic field  $\vec{B}$  gives rise to induced electromagnetic force referred to as Lorentz's force written as:

$$\vec{F} = \vec{j} \times \vec{B}. \quad (3)$$

Maxwell's equations are:

$$\vec{\nabla} \cdot \vec{B} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \vec{\nabla} \times \vec{B} = \mu_0 \vec{j}, \vec{\nabla} \cdot \vec{E} = \frac{\rho_s}{\epsilon_0} \quad (4)$$

The generalized Ohm's law is:

$$\vec{j} = \sigma(\vec{E} + \vec{U} \times \vec{B}) \quad (5)$$

The body forces considered is electromagnetic force and gravitational forces are given as:

$$\vec{F}_b = \rho g + \vec{j} \times \vec{B} \quad (6)$$

The magnetic component are given as  $B_x = B_o \cos \alpha$ ,  $B_y = 0$  and  $B_z = 0$ . Thus, the magnetic distribution is written as:

$$\vec{B} = \langle B_o \cos \alpha, 0, 0 \rangle \quad (7)$$

The velocity distribution of blood is given as:  $\vec{U} = \langle 0, 0, w(x, t) \rangle$  (8)

Thus, Lorentz's force  $\vec{j} \times \vec{B}$  is calculated as:

$$\vec{j} = \sigma(\vec{w} \times \vec{B}) = \sigma \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & w \\ B_o \cos \alpha & 0 & 0 \end{bmatrix} = \sigma B_o w \cos(\alpha) \hat{j} \quad (9)$$

$$\vec{j} \times \vec{B} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \sigma B_o w \cos \alpha & 0 \\ B_o \cos \alpha & 0 & 0 \end{bmatrix} = -\sigma B_o^2 w \cos^2(\alpha) \hat{k} \quad (10)$$

Taking the angle of inclination of the tube into consideration, the momentum equation for the blood flow in equation is written as:

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial x^2} + \rho g_z \sin \theta - \sigma B_o^2 w \cos^2 \alpha \quad (11)$$

Since  $w$  is independent of  $z$ , this makes  $[(\vec{w} \cdot \nabla) \vec{w}]$  in the Navier-Stokes equation (11) vanish. Thus, the momentum equation for this flow reduces to:

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial x^2} + \rho g_z \sin \theta - \sigma B_o^2 w \cos^2 \alpha \quad (12)$$

The initial conditions for the flow are written as:

$$w(x, 0) = 0 \text{ at } t = 0 \quad (13)$$

The boundary conditions for flow are given as:

$$w(x_o, t) = 0 \text{ on } x = x_o \quad (\text{No slip condition}) \quad (14a)$$

$$w(x, t) = U_o \text{ is finite at axis of the tube, } x = 0 \quad (14b)$$

$$w(-x_o, t) = 0 \text{ on } x = -x_o \quad (\text{No slip condition}) \quad (14c)$$

Introducing scaling variables and dimensionless parameters given below:

$$t^* = \frac{tw_0}{L}, P^* = \frac{PL}{\mu w_0}, W^* = \frac{w}{w_0}, x^* = \frac{x}{L}, Z^* = \frac{Z}{L}, Re = \frac{\rho L w_0}{\mu}, Fr = \frac{w_0^2}{gL}, Ha^2 = \frac{\sigma L^2 B_0^2}{\mu} \tag{15}$$

The continuity and momentum equations in non-dimensionalized form are written as:

$$\frac{\partial w^*}{\partial z^*} = 0 \tag{16}$$

$$Re \frac{\partial w^*}{\partial t^*} = -\frac{\partial P^*}{\partial Z^*} + \frac{\partial^2 w^*}{\partial x^{*2}} + \frac{Re}{Fr} \sin\theta - Ha^2 w^* \cos^2\alpha \tag{17}$$

The pressure gradient in dimensionless form is taken as:  $-\frac{\partial p}{\partial z} = G_0 + G_1 \cos\omega t \ t \geq 0$

In which  $G_0$  is the steady state part of pressure gradient,  $G_1$  is the amplitude of the pulsatile component giving rise to systolic and diastolic pressure and part,  $\omega = 2\pi f$ , where  $f$  is the pulse frequency.

### 3. The Solution Procedure

Writing the momentum equation in finite difference form it becomes:

$$Re \left( \frac{w_i^{j+1} - w_i^j}{\Delta t} \right) = -\frac{\partial P^*}{\partial Z^*} + \left\{ \frac{1}{2} \left( \frac{w_{i+1}^j - 2w_i^j + w_{i-1}^j}{(\Delta x)^2} + \frac{w_{i+1}^{j+1} - 2w_i^{j+1} + w_{i-1}^{j+1}}{(\Delta x)^2} \right) \right\} + \frac{Re}{Fr} \sin\theta - Ha^2 \left\{ \frac{1}{2} (w_i^j + w_{i+1}^j) \cos 2\alpha \right. \tag{18}$$

Arranging the equation and dropping the asterisk (\*) yields:

$$\left( -\frac{1}{2(\Delta x)^2} \right) w_{i-1}^{j+1} + \left( \frac{Re}{\Delta t} + \frac{1}{(\Delta x)^2} + \frac{Ha^2 \cos^2\alpha}{2} \right) w_i^{j+1} + \left( -\frac{1}{2(\Delta x)^2} \right) w_{i+1}^{j+1} = \left( \frac{1}{2(\Delta x)^2} \right) w_{i-1}^j + \left( \frac{Re}{\Delta t} - \frac{1}{(\Delta x)^2} - \frac{Ha^2 \cos^2\alpha}{2} \right) w_i^j + \left( \frac{1}{2(\Delta x)^2} \right) w_{i+1}^j + \frac{Re}{Fr} \sin\theta + G_0 + G_1 \cos\omega t \tag{19}$$

where for  $i = 1, 2, 3, \dots, n$ .

$$\text{Let } A_i = -\frac{1}{2(\Delta x)^2}, B_i = \frac{1}{(\Delta x)^2} + \frac{Ha^2 \cos^2\alpha}{2} \text{ and } C_i = -\frac{1}{2(\Delta x)^2} \tag{20a}$$

the equation (19) yields:

$$A_i w_{i-1}^{j+1} + (B_i + \frac{Re}{\Delta t}) w_i^{j+1} + C_i w_{i+1}^{j+1} = -A_i w_{i-1}^j + (\frac{Re}{\Delta t} - B_i) w_i^j - C_i w_{i+1}^j + \frac{Re}{Fr} \sin\theta + G_0 + G_1 \cos\omega t \tag{20b}$$

For  $i = 1$ : Let  $\frac{Re}{\Delta t} + B_i = \overline{B}_1$ ,  $\frac{Re}{\Delta t} - B_i = \overline{\overline{B}}_1$  and  $\frac{Re}{Fr} \sin\theta + G_0 + G_1 \cos\omega t = D_1$ , then equation (20b) becomes:

$$A_1 w_0^{j+1} + \overline{B}_1 w_1^{j+1} + C_1 w_2^{j+1} = -A_1 w_0^j + \overline{\overline{B}}_1 w_1^j - C_1 w_2^j + D_1 \tag{21a}$$

For  $i = 2$ :

$$\begin{matrix} A_2 w_1^{j+1} + \overline{B}_2 w_2^{j+1} + C_2 w_3^{j+1} = -A_2 w_1^j + \overline{\overline{B}}_2 w_2^j - C_2 w_3^j + D_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{matrix} \tag{21b}$$

For  $i = n - 1$ :

$$A_{n-1} w_{n-2}^{j+1} + \overline{B}_{n-1} w_{n-1}^{j+1} + C_{n-1} w_n^{j+1} = -A_{n-1} w_{n-2}^j + \overline{\overline{B}}_{n-1} w_{n-1}^j - C_{n-1} w_n^j + D_{n-1} \tag{21c}$$

The algebraic system of equations above (21a), (21b) and (21c) can be represented in a tri-diagonal matrix form as:

$$\begin{bmatrix} A_1 \overline{B_1} C_1 & 0 & 0 & \dots & 0 \\ 0 & A_2 \overline{B_2} C_2 & 0 & \dots & 0 \\ 0 & 0 & A_3 \overline{B_3} C_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_{n-2} \overline{B_{n-2}} C_{n-2} & 0 \\ 0 & 0 & \dots & A_{n-1} \overline{B_{n-1}} C_{n-1} & 0 \end{bmatrix} \begin{bmatrix} w_0^{j+1} \\ w_1^{j+1} \\ w_2^{j+1} \\ \vdots \\ w_{n-1}^{j+1} \\ w_n^{j+1} \end{bmatrix} = \begin{bmatrix} A_1 \overline{B_1} C_1 & 0 & 0 & \dots & 0 \\ 0 & A_2 \overline{B_2} C_2 & 0 & \dots & 0 \\ 0 & 0 & A_3 \overline{B_3} C_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & A_{n-2} \overline{B_{n-2}} C_{n-2} & 0 \\ 0 & 0 & \dots & A_{n-1} \overline{B_{n-1}} C_{n-1} & 0 \end{bmatrix} \begin{bmatrix} w_0^j \\ w_1^j \\ w_2^j \\ \vdots \\ w_{n-1}^j \\ w_n^j \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_{n-1} \\ D_n \end{bmatrix} \quad (22)$$

The equations are subjected to initial and boundary conditions

$$w(x, 0) = 0, w(1, t) = 0, w(0, t) = 1, w(-1, t) = 0 \quad (23)$$

### 4. Results and Discussion

A glimpse on the outlook of the numerical solutions for velocity profiles using the given reference values  $G_0 = 0.2, G_1 = 0.4, Fr = 0.05, Re = 0.4, f = 1.2\text{Hz}, \omega = 2.4\pi$  and  $Ha = 0, 1, 2$  and 3. The relationship of  $\theta$  and  $\alpha$  was given by  $\alpha = 90 - \theta$ . The output graphs are provided below.

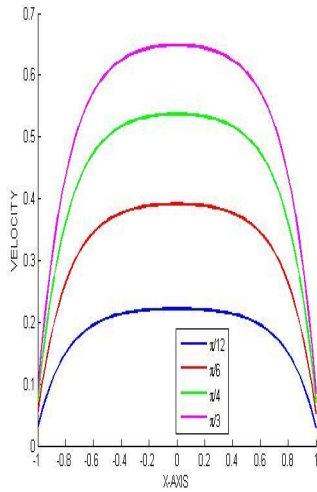


Fig. 1. Velocity profiles for Ha=0

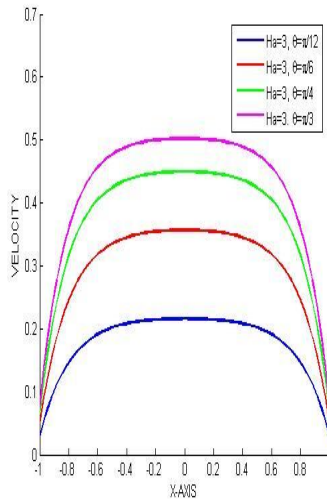


Fig. 2. Velocity profiles for Ha=3

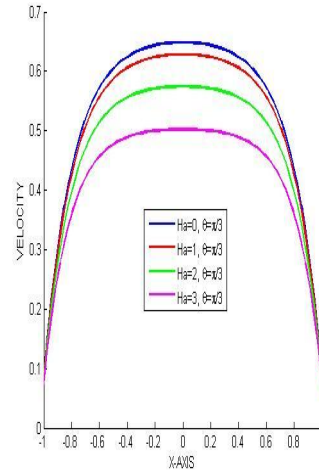


Fig. 3. Velocity profiles for varying Ha [  $\theta = \pi/3, \alpha = \pi/6$  ]

In Fig. 1 it is observed that when the angle of inclination of the tube is increased while Re and Fr are held constant, the magnitude of velocity profile increases with maximum velocity attained when  $x = 0$  and becomes zero at the walls of the tube. This can be explained as the inclination angle of the tube increases, the axial velocity of the blood flow in the tube increases due to gravitational acceleration and velocity of blood at the wall is the same as the velocity of tube due to no slip condition.

In Fig. 2 it is observed that the angle of inclination of the tube is varied with time, with Hartmann number  $Ha=3$  kept constant. Increase in the angle of inclination of the tube increases the axial velocity of the blood flow. The velocity profiles appeared very blunt due to influence of strong magnetic field. This is because



increasing the inclination angle of tube accelerated the flow of blood due to earth's gravitational force. Consequently, high Lorentz's force acts on the constituent particles of blood because of a strong magnetic field thus decelerating the axial velocity of flow. The magnitude of Lorentz's force experienced in the main flow rely on strength of magnetic field and the velocity of electrically conducting blood.

In Fig. 3 when the tube is inclined at an angle  $\theta = \pi/3$  and strong magnetic field inclined at an angle  $\alpha = \pi/6$  the axial velocity is found to be maximum in absence of magnetic field. When the angle of inclination of the tube is enlarged, the blood flows with high velocity due to increased acceleration from earth's gravitational force. Increasing the Hartmann number reduced the axial velocity of blood because of high Lorentz's force produced thus retarding the flow. The bluntness of velocity profile is quite significant and decreases as the magnetic field is increased because of more resistive force produced. The magnitude of resistive Lorentz's force depends on the motion of the blood flow and strength of magnetic field.

## 5. Conclusion

From the discussion, it is noticed that magnetic field affect largely the axial velocity of blood flow as well as increasing the inclination angle of the tube. Therefore, a remarkable phenomenon is that on using appropriate external magnetic field we can regulate the axial velocity of the blood flow.

## Acknowledgements

The authors gratefully acknowledge the department of Mathematics of Meru University of Science and Technology for their constant support and encouragement and referees for the valuable suggestions that led to the improvement of this paper.

## Competing Interests

Authors have declared that no competing interests exist.

## References

- [1] Kolin A. Electromagnetic flow meter: Principle of method and its applications to blood flow measurement. *Journal in Proceedings of the Society for Experiment of Biology and Medicine*. 1936;35:53-56.
- [2] Korchevskii EM, Marochnik LS. Magneto hydrodynamic version of movement of blood. *Journal of biophysics*. 1965;10(2):411- 414.
- [3] Vardanyan VA. Effect of magnetic field on blood flow. *Biophysics Journal*. 1973;18:515.
- [4] Sud VK, Suri PK, Mishra RK. Laminar flow of blood in an elastic tube in the presence of magnetic field. *Studia Biophysics*. 1978;69.
- [5] Suri PK. Effect of static magnetic field on blood flow in a branch. *Indian Journal Pure Appl. Math*. 1981;12(7):907-908.
- [6] Sanyal DC, Das K, Debnath S. Effect of magnetic field on pulsatile blood flow through an inclined circular tube with periodic body acceleration. *Journal of Physical Science*. 2007;11:43-56.
- [7] Dar AA, Elangovan K. Influence of an Inclined Magnetic Field and Rotation on the Peristaltic Flow of a Micropolar Fluid in an Inclined Channel. *New Journal of Science*. 2016;3(4):1-14.

- [8] Bali R, Awasthi U. Mathematical model of blood flow in small blood vessel in the presence of magnetic field. *Applied Mathematics*. 2011;2:264-269.
- [9] Tanwar VK, Varshney NK, Agarwal R. The effect of porous medium on pulsatile inclined two layered blood flows with magnetic field and periodic body acceleration. *IOSR Journal of Mathematics*. 2014;10(1):1-14.
- [10] Tripathi D. A mathematical model for blood flow through inclined arteries under the influence of inclined magnetic field. *Journal of Mechanic in Medicine and Biology*. 2012;12(3 1250033):1-16.
- [11] Shit GC. Computational modelling of blood flow development and its characteristics in magnetic environment. Hindawi Publishing Corporation *Modelling and Simulation in Engineering*, 2013;758748-758760.
- [12] Sharma MK, Singh K, Bansal S. Pulsatile MHD flow in a catheterized Stenosed artery with slip on the wall. *Journal of Biomedical Sciences and Engineering*. 2014;7:194-207.
- [13] Srivastava N. Analysis of flow characteristics of the blood flowing through an inclined tapered porous artery with mild stenosis under the influence of an inclined magnetic field. *Journal of Biophysics*. 2014;(797142):1-9.
- [14] Kumar A, Chandel RS, Shrivastava R, Shrivastava K, Kumar S. Mathematical modeling of blood flow in an inclined tapered artery under MHD effect through porous medium. *International Journal of Pure and Applied Mathematical Sciences*. 2016;9(1):75-88.
- [15] Wahab MA, Salem SI. Magneto hydrodynamic blood flow in a narrow tube. *World Research Journal of Biomaterials*. 2012;1(1):1-7.
- [16] Jyothi S, Reddy MVS, Prasad R. Effect of magnetic field on the peristaltic flow of a Newtonian fluid in an inclined tube with variable viscosity. *International Journals of Conceptions on Computing and Information Technology*. 2014;2(1):32-37.
- [17] Dar AA, Elangovan K. Influence of an inclined magnetic field on Heat and Mass transfer of the peristaltic flow of a couple stress fluid in an inclined channel. *World Journal of Engineering*. 2017;14(1):7-18.
- [18] Dar AA, Elangovan K. Impact of an inclined magnetic field, heat generation or absorption and radiation on the peristaltic flow of a Micropolar fluid through a porous non-uniform channel with slip velocity. *New Trends in Mathematical Sciences*. 2017;5(3):227-244.
- [19] Verma SR, Srivastava A. Effect of magnetic field on steady blood flow through an inclined circular tube. *International Journal of Engineering Research and Application*. 2013;3(4):428-432.

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