British Journal of Mathematics & Computer Science
2(3): 126-136, 2012

SCIENCEDOMAIN *international*
www.sciencedomain.org

SCIENCEDOMAIN

A Food Chain Model with Ratio-dependent Functional Response, Impulsive Perturbations and Feedback Controls

Jian-wen Jia[1*] and Jian-feng Liu[1]

[1] *School of Mathematics and Computer Science, Shanxi Normal University , Shanxi, Linfen, 041004, China*

Research Article

Received: 27 February, 2012
Accepted: 30 March, 2012
Published: 13 October, 2012

Abstract

In this paper, a food chain system with ratio-dependent functional response, impulses,feedback controls and delays is studied. By using the theorem of coincidence degree, homotopy invariance property and Lyapunov's approach, a set of sufficient conditions for ensuring the existence and stability of positive periodic solutions of the system are derived. The results extend some recent works.

Keywords: Predator-prey system; Feedback controls ; Impulses; Delays ; Periodic solution ; Coincidence degree

2010 Mathematics Subject Classification: 34C25;92B25

1 Introduction

Predator-prey system is the classic model in ecology, has been studied extensively (Cheng and Li, 2007; Li et al., 2003). At present, two species predator-prey system with feedback control models have become hot, also systems which based on rate-dependent functional response have also been received much attention. The ratio-dependent functional response is the density ratio function on predator and prey two groups. Generally, predator-prey system with ratio-dependent functional response is

$$\begin{cases} \frac{dx}{dt} = f(x) - yp(\frac{x}{y}), \\ \frac{dy}{dt} = kyq(\frac{x}{y}) - dy. \end{cases} \tag{1}$$

The literature [Chen et al.(2003); Gopalsamy and Wang (1993)] proposed the following feedback system

$$\begin{cases} \frac{dn(t)}{dt} = n(t)[1 - \frac{a_1 n(t)+a_2 n(t-\tau)}{k} - cv(t)], \\ \frac{dv(t)}{dt} = -av(t) + bn(t), \end{cases} \tag{2}$$

*Corresponding author: E-mail: jiajw.2008@163.com

Here $n(t)$ and $v(t)$ stand for population density of the specie and the feedback controls variable, respectively. The paper studied the stability of positive equilibrium point, and also gave the global asymptotic stability conditions.

The literature (Li and Wang, 2009) studied a food chain system with ratio-dependent functional response, delays and feedback controls to advanced predator. The model as follows

$$\begin{cases} \frac{dx_1(t)}{dt} = x_1(t)[a_1(t) - b_1(t)x_1(t) - \frac{c_1(t)x_2(t)}{m_1x_2(t)+x_1(t)}], \\ \frac{dx_2(t)}{dt} = x_2(t)[-a_2(t) + \frac{f_1(t)x_1(t-\tau_1)}{m_1x_2(t-\tau_1)+x_1(t-\tau_1)} - \frac{c_2(t)x_3(t)}{m_2x_3(t)+x_2(t)}], \\ \frac{dx_3(t)}{dt} = x_3(t)[-a_3(t) + \frac{f_2(t)x_2(t-\tau_2)}{m_2x_3(t-\tau_2)+x_2(t-\tau_2)}] - c_3(t)v(t)x_3(t), \\ \frac{dv(t)}{dt} = -a(t)v(t) + b(t)x_3(t). \end{cases} \tag{3}$$

The author investigated the existence of periodic solutions.

The literatures (Li and Wang, 2009; Si and Chen, 2007) introduces feedback control variables to the three groups, respectively; literature (Yang and Xu, 2009), proposes a food chain system, in order to close to the actual situation introduces pulse effects. In this paper, basing on model (1), (2) and (3), introduces feedback control variables for the three groups respectively and pulse interference.

2 Model formulation

This paper, basing on model (1), (2) and (3), introduces feedback control variables for the three groups respectively and pulse interference, thus the following food chain system was established.

$$\begin{cases} \left.\begin{array}{rl} \frac{\mathrm{d}x_1(t)}{\mathrm{d}t} = & x_1(t)[a_1(t) - b_1(t)x_1(t) - \frac{c_1(t)x_2(t)}{m_1(t)x_2(t)+x_1(t)}] - h_1(t)v_1(t)x_1(t) \\ \frac{\mathrm{d}x_2(t)}{\mathrm{d}t} = & x_2(t)[-a_2(t) + \frac{f_1(t)x_1(t-\tau_1)}{m_1(t)x_2(t-\tau_1)+x_1(t-\tau_1)} - \frac{c_2(t)x_3(t)}{m_2(t)x_3(t)+x_2(t)}] \\ & -h_2(t)v_2(t)x_2(t) \\ \frac{\mathrm{d}x_3(t)}{\mathrm{d}t} = & x_3(t)[-a_3(t) + \frac{f_2(t)x_2(t-\tau_2)}{m_2(t)x_3(t-\tau_2)+x_2(t-\tau_2)}] - h_3(t)v_3(t)x_3(t) \\ \frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = & -\alpha_i(t)v_i(t) + \beta_i(t)x_i(t) \quad i = 1,2,3. \end{array}\right\} \quad t \neq t_k, \\ \Delta x_i = x_i(t_k^+) - x_i(t_k) = d_{ik}x_i(t_k) \qquad i = 1,2,3. \quad k \in N. \end{cases} \tag{4}$$

Here $x_i(t)$ denote the population density of the i specie in this food chain at time t, $v_i(t)$ stand for the feedback control variables for the i specie, $(i = 1, 2, 3)$, Δx_i is incremental of the corresponding population at the pulse time $t = t_k$.

The ecological significance for system (4): because of birth, acquisition, stocking and other transient factors, the three groups is in the case of high growth rates, through the introduction the feedback control variables to the three groups, we come to control the growth rates of three species, achieve to the overall controls for the system, and keep the ecosystem balance. We noted that the feedback controls variables represent the interference which people have done, so the system (4) has much practical value. This paper studies existence and global asymptotic stability of Positive Periodic Solutions of system (4).

Note: This article discusses the generally model. When $\Delta x_i = 0$, the system(4) corresponds to feedback controls predator-prey systems without pulse, such as (Si and Chen, 2007) studied the persistence of the system, and obtained sufficient conditions for existence of periodic solutions.

When $\alpha_i(t) = 0$, the system(4)corresponds to the pulse type predator-prey system without feedback controls, such as (Zhang et al., 2005) studied local stability of the predator eradication periodic solution of the the system.

When $\Delta x_i = 0, \alpha_i(t) = 0$, the system(4) corresponds to predator-prey system with no pulse and no feedback controls, such as (Xu and Chen, 2001) studied the persistence of the system and global asymptotic stability.

For the sake of generality and convenience, we make the following fundamental assumptions for system (4):

(H1) $\{t_k\}$ satisfies: $t_k < t_{k+1}$ and $\lim\limits_{k\to\infty} t_k = \infty, \quad k \in N$;

(H2) There exists an integer $\omega > 0$ and positive integer p, such that $t_{kp} = t_k + \omega, d_{i,k+p} = d_{ik} \geq 0. \quad k \in N, \quad i = 1, 2, 3$;

(H3) $a_i(t), h_i(t), \alpha_i, \beta_i, i = 1, 2, 3; f_i(t), c_i(t), m_i(t), i = 1, 2$ and $b_1(t)$ is a nonnegative continuous $\omega-$Periodic function, τ_1, τ_2 is a nonnegative constants.

For continuous $\omega-$ periodic function $g(t)$ and $\omega-$ periodic sequence $\{d_{ik}\}$, set

$$g^l = \inf_{t\in[0,\omega]} g(t), \qquad g^u = \sup_{t\in[0,\omega]} g(t), \qquad \bar{g} = \frac{\int_0^\omega g(t)dt}{\omega},$$

$$|\ \bar{g}\ | = \frac{\int_0^\omega |\ g(t)\ |\ dt}{\omega}, \qquad \overline{\Delta}_i = \frac{\sum\limits_{k=1}^{p} \ln(1 + d_{ik})}{\omega}.$$

Considering the initial conditions

$$x_i(s) = \phi_i(s), s \in [-\tau, 0], x_i(0) > 0,$$

$$v(t) = \varphi_i(t), t \in [0, \omega], v_i(0) > 0, i = 1, 2, 3;$$

and $\tau = \max\{\tau_1, \tau_2\}$.

2.1 Important lemma

In order to study the existence of periodic solutions of the system (4), we use the following definition and Lemma[10].

Let X, Z be real Banach spaces, $L : \mathrm{DomL} \subset X \longrightarrow Z$ is the linear mapping, $\ker L = L^{-1}(0)$ is the nucleus of L, $\mathrm{Im}L = L(\mathrm{Dom}L)$ is the range of L,I is the identity mapping.

The mapping L is said to be a Fredholm mapping of index zero, if $\dim\ker L = \mathrm{codimIm}L < +\infty$, and $\mathrm{Im}L$ is closed in Z.

If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \longrightarrow X$ and $Q : Z \longrightarrow Z$, such that $\mathrm{Im}p = \ker L, \mathrm{Im}L = \ker Q = \mathrm{Im}(I - Q)$, It follows that the restriction L_p of L to $\mathrm{Dom}L \bigcap \ker P$: $(I - P)X \longrightarrow \mathrm{Im}L$ is invertible. Denote the inverse of L_P by K_P, $K_P : \mathrm{Im}L = \mathrm{Dom}L$, then $PK_p = 0, LK_p\ |_{\mathrm{Im}L} = I, K_pL\ |_{\mathrm{Dom}L} = I - P$.

The mapping $N : X \to Z$ is said to be $L-$ compact on $\overline{\Omega}$, if Ω is an open bounded subset of X, $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \longrightarrow X$ is compact.

Since $\mathrm{Im}Q$ is isomorphic to $\ker L$, there exists an isomorphism $J : \mathrm{Im}Q \longrightarrow \ker L$.

Lemma 1 $(YangandXu, 2009)$ Let $\Omega \subset X$ be an open bounded set, L be a Fredholm mapping of index zero and $N : \overline{\Omega} \to Z$ be $L-$ compact on $\overline{\Omega}$. Assume

(1) for each $\lambda \in (0, 1)$, $x \in \partial\Omega \bigcap \mathrm{Dom}L$, $Lx \neq \lambda Nx$;

(2) for each $x \in \partial\Omega \bigcap \ker L$, $QNx \neq 0$;

(3) $\deg\{JQN, \Omega \bigcap \ker L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in $\mathrm{Dom}L \bigcap \overline{\Omega}$.

Lemma 2 $(Gopalsamy, 1992)$ A non-negative function $f(t) \in PC[[0, \infty), R]$, and $\int_0^{+\infty} f(s)ds < +\infty$. if for each $\epsilon > 0$ and $n \in N$, there exists $\delta > 0$, when $s_1, s_2 \in (t_{n-1}, t_n], |s_1 - s_2| < \delta$, there is $|f(s_1) - f(s_2)| < \epsilon$, then $\lim\limits_{t\to+\infty} f(t) = 0$.

Lemma 3 $R_+^6 = \{(x_1(t), x_2(t), x_3(t), v_1(t), v_2(t), v_3(t))^T \in R^6 : x_i(t) > 0, v_i(t) > 0, i = 1, 2, 3\}$ is the positive invariant set of system (4).

Proof. For $t \in (t_{k-1}, t_k]$, by the first and fifth equations of system (4), we have

$$x_1(t) = x_1(0)(1 + d_{1k})^{k-1} \exp\{\int_0^t [a_1(s) - b_1(s)x_1(s) - \tfrac{c_1(s)x_2(s)}{m_1x_2(s)+x_1(s)} - h_1(t)v_1(t)]ds\},$$

because of the initial conditions $x_i(0) > 0$, $x_1(t) > 0$ is clear.

Similarly, by the second, third and fifth equations of system (4), we have

$$x_2(t) \geq x_2(0)(1+d_{2k})^{k-1}\exp\{\int_0^t[-a_2(s) - \frac{c_2(s)x_3(s)}{m_2x_3(s)+x_2(s)} - h_2(s)v_2(s)]ds\},$$
$$x_3(t) \geq x_3(0)(1+d_{3k})^{k-1}\exp\{\int_0^t[-a_3(s) - h_3(s)v_3(s)]ds\},$$

because of the initial conditions $x_2(0) > 0, x_3(0) > 0$,$x_2(t) > 0$ and $x_3(t) > 0$ is clear.

In addition, from the forth equations of the system (4), we have

$$v_i(t) = e^{-\int_0^t \alpha_i(s)ds}[v_i(0) + \int_0^t \beta_i(s)x_i(s)e^{\int_0^s \alpha_i(u)du}ds],$$

Therefore,because of the initial conditions $v_i(0) > 0$, $\beta_i(t) \geq 0$ and $x_i(t) > 0 (i = 1, 2, 3)$, we obtain $v_i(t) > 0 (i = 1, 2, 3)$. This completes the proof.

3 Existence of positive periodic solutions

Theorem 1 In addition to(H1)-(H3),assume further that

(1) $\bar{a}_1 > \frac{\bar{c}_1}{m_1^l} + \frac{\bar{h}_1\bar{\beta}_1\bar{a}_1}{\bar{b}_1}e^{\omega(\overline{\Delta}_1+2\bar{a}_1)}(\frac{1}{\bar{\alpha}_1} + 2\omega)$;

(2) $\bar{f}_1 > \bar{a}_2 + \frac{\bar{c}_2}{m_2^l} + \frac{\bar{h}_2\bar{\beta}_2\bar{a}_1\bar{f}_1}{\bar{b}_1\bar{a}_2m_1^l}e^{\omega(\overline{\Delta}_1+\overline{\Delta}_2+2\bar{a}_1+2\bar{f}_1)}(\frac{1}{\bar{\alpha}_2} + 2\omega)$;

(3) $\bar{f}_2 > \bar{a}_3 + \frac{\bar{h}_3\bar{\beta}_3\bar{a}_1\bar{f}_1\bar{f}_2}{\bar{b}_1\bar{a}_2\bar{a}_3m_1^lm_2^l}e^{\omega(\overline{\Delta}_1+\overline{\Delta}_2+\overline{\Delta}_3+2\bar{a}_1+2\bar{f}_1+2\bar{f}_2)}(\frac{1}{\bar{\alpha}_3} + 2\omega)$.

Then system (4) has at least one positive ω periodic solution.

Proof. Let $x_i(t) = e^{y_i(t)}, \quad i = 1, 2, \ldots, n.$ It follows that

$$\left\{\begin{array}{l} \left.\begin{array}{rcl} \frac{dy_1(t)}{dt} &=& a_1(t) - b_1(t)e^{y_1(t)} - \frac{c_1(t)e^{y_2(t)}}{m_1(t)e^{y_2(t)}+e^{y_1(t)}} - h_1(t)v_1(t), \\ \frac{dy_2(t)}{dt} &=& -a_2(t) + \frac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)}+e^{y_1(t-\tau_1)}} \\ && -\frac{c_2(t)e^{y_3(t)}}{m_2(t)e^{y_3(t)}+e^{y_2(t)}} - h_2(t)v_2(t), \\ \frac{dy_3(t)}{dt} &=& -a_3(t) + \frac{f_2(t)e^{y_2(t-\tau_2)}}{m_2(t)e^{y_3(t-\tau_2)}+e^{y_2(t-\tau_2)}} - h_3(t)v_3(t), \\ \frac{dv_i(t)}{dt} &=& -\alpha_i(t)v_i(t) + \beta_i(t)e^{y_i(t)} \quad i = 1, 2, 3. \end{array}\right\} \quad t \neq t_k, \\ \Delta y_i = y_i(t_k^+) - y_i(t_k) = \ln(1 + d_{ik}) \qquad i = 1, 2, 3. \quad k \in N. \end{array}\right. \tag{5}$$

Set $y(t) = (y_1(t), y_2(t), y_3(t))^T, v(t) = (v_1(t), v_2(t), v_3(t))^T$ .Let

$Y = \{U(t) = (y(t)^T, v(t)^T)^T \in PC(R, R_+^6)|U(t+\omega) = U(t)\}$,

$Z = Y \times R^{6p} = \{z = (((y(t))^T, (v(t))^T)^T, \{(\ln(1+d_{1k}), \ln(1+d_{2k}), \ln(1+d_{3k}))^T, 0, 0, 0\} |_1^p |k = 1, 2, \ldots, p\}$,

$\|U\|_Y = \sup\limits_{t\in[0,\omega]} \|y(t)\| + \sup\limits_{t\in[0,\omega]} \|v(t)\|, \qquad \|V\|_Z = \|U\|_Y + \|z\|,$ here $U \in Y, V = (U, z) \in Z, z \in R_+^{6p}$,$\|\cdot\|$ is the corresponding norm in R^{6p},then Y, Z is a Banach spaces. Let $L : \mathrm{Dom}L \longrightarrow Z$ and $N : Y \longrightarrow Z$,

$$LU = (U'(t), \Delta U(t_1), \ldots, \Delta U(t_p)),$$

where $U \in \mathrm{Dom}L$, $\qquad \Delta U(t_k) = \begin{pmatrix} y(t_k^+) - y(t_k) \\ 0 \end{pmatrix},$

and $\mathrm{Dom}L = \{U(t)|U(t) = (y(t), v(t))^T \in Y \bigcap PC^1(R, R^6)\}$.

$$NU = (\Phi(t), \Xi_1, \ldots, \Xi_p),$$

where $\Phi(t) = (\phi_1(t), \phi_2(t), \phi_3(t), \varphi_1(t), \varphi_2(t), \varphi_3(t))^T$,

$\Xi_l = (\ln(1 + d_{1l}, \ln(1 + d_{2l}), \ln(1 + d_{3l}), 0, 0, 0) \in R^6, \quad l = 1, 2, \ldots, p.$

and $\phi_1(t) = a_1(t) - b_1(t)e^{y_1(t)} - \frac{c_1(t)e^{y_2(t)}}{m_1(t)e^{y_2(t)}+e^{y_1(t)}} - h_1(t)v_1(t)$,
$\phi_2(t) = -a_2(t) + \frac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)}+e^{y_1(t-\tau_1)}} - \frac{c_2(t)e^{y_3(t)}}{m_2(t)e^{y_3(t)}+e^{y_2(t)}} - h_2(t)v_2(t)$,
$\phi_3(t) = -a_3(t) + \frac{f_2(t)e^{y_2(t-\tau_2)}}{m_2(t)e^{y_3(t-\tau_2)}+e^{y_2(t-\tau_2)}} - h_3(t)v_3(t)$,
$\varphi_i(t) = -\alpha_i(t)v_i(t) + \beta_i(t)e^{y_i(t)} \quad i = 1, 2, 3.$
Then

$$\mathrm{ker}L = R^6, \ \ \mathrm{Im}L = \{V = (U, z_1, \dots, z_p) \in Z | \int_0^\omega U(s)ds + \sum_{j=1}^p z_p = 0\}.$$

so $\dim \mathrm{ker}L = \mathrm{CodimIm}L < +\infty$ and $\mathrm{Im}L$ is closed in Z. Therefore L is a Fredholm mapping of index zero.As to $U \in Y, V = (U, z_1, \dots, z_p) \in Z$, define two projectors $P : Y \longrightarrow Y$ and $V : Z \longrightarrow Z$ as

$$PU = \frac{1}{\omega}\int_0^\omega U(t)dt, \qquad QV = (\frac{1}{\omega}(\int_0^\omega U(t)dt + \sum_{j=1}^p z_p), 0, \dots, 0),$$

then P and Q are continuous projectors such that $\mathrm{Im}P = \ker L$, and $\mathrm{Im}L = \ker Q = \mathrm{Im}(I - Q)$. Furthermore, through an easy computation we find that the inverse K_p of $L|_{\mathrm{Dom}L \bigcap \ker P}$: $(I - P)X \longrightarrow \mathrm{Im}L$, has the form

$$K_P : \mathrm{Im}L \longrightarrow \mathrm{Dom}L \bigcap \ker P.$$

For each $V = (U, z_1, \dots, z_p) \in Z$, there exists $\chi \in X$ such that $\chi'(t) = U(t), t \neq t_k, k \in N, U(t_k^+) - U(t_k) = z_k$,then $\chi(t) = \int_0^t U(s)ds + \sum\limits_{t>t_k} z_k + U(0)$. And $\int_0^\omega \chi(t)dt = 0$,for $\chi \in \ker P$, such that

$$\int_0^\omega \int_0^t U(s)dsdt + \int_0^\omega \sum_{t>t_k} z_k dt + \omega U(0) = 0.$$

Then

$$K_{pz} = \chi(t) = \int_0^t U(s)ds + \sum_{t>t_k} z_k - \frac{1}{\omega}\int_0^\omega \int_0^t U(s)dsdt - \frac{1}{\omega}\sum_{j=1}^p (\omega - t_k)z_k.$$

Clearly, QN and $K_p(I - Q)N$ are continuous.By using Arzela-Ascoli Theorem, it is not difficult to prove that $QN(\bar{\Omega})$ and $K_p(I - Q)N(\bar{\Omega})$ are relatively compact for any open bounded set $\Omega \in Y$. Therefore $N : Y \longrightarrow Z$ is $L-$compact on $\bar{\Omega}$.

In order to apply Lemma 2.2, we need to find an appropriate open, bounded subsets Ω in Y. Assume $\Omega = \{U | \|U\| < H\}$, here H is a constant to be determined. Corresponding to the operator equation $LU = \lambda NU, U \in Y$,$\lambda \in (0, 1)$, we have

$$\left\{\begin{array}{rl}\frac{\mathrm{d}y_1(t)}{\mathrm{d}t} = & \lambda[a_1(t) - b_1(t)e^{y_1(t)} - \frac{c_1(t)e^{y_2(t)}}{m_1(t)e^{y_2(t)}+e^{y_1(t)}} - h_1(t)v_1(t)], \\ \frac{\mathrm{d}y_2(t)}{\mathrm{d}t} = & \lambda[-a_2(t) + \frac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)}+e^{y_1(t-\tau_1)}} \\ & -\frac{c_2(t)e^{y_3(t)}}{m_2(t)e^{y_3(t)}+e^{y_2(t)}} - h_2(t)v_2(t)], \\ \frac{\mathrm{d}y_3(t)}{\mathrm{d}t} = & \lambda[-a_3(t) + \frac{f_2(t)e^{y_2(t-\tau_2)}}{m_2(t)e^{y_3(t-\tau_2)}+e^{y_2(t-\tau_2)}} - h_3(t)v_3(t)], \\ \frac{\mathrm{d}v_i(t)}{\mathrm{d}t} = & \lambda[-\alpha_i(t)v_i(t) + \beta_i(t)e^{y_i(t)}], \end{array}\right\} \quad t \neq t_k, \qquad (6)$$
$$\Delta y_i = y_i(t_k^+) - y_i(t_k) = \lambda \ln(1 + d_{ik}) \qquad i = 1, 2, 3. \quad k \in N.$$

Integrating (6) over the interval $[0, \omega]$ leads to

$$\int_0^\omega [b_1(t)e^{y_1(t)} + \frac{c_1(t)e^{y_2(t)}}{m_1(t)e^{y_2(t)} + e^{y_1(t)}} + h_1(t)v_1(t)]dt = \bar{a}_1\omega, \qquad (7)$$

$$\int_0^\omega [a_2(t) + \frac{c_2(t)e^{y_3(t)}}{m_2(t)e^{y_3(t)} + e^{y_2(t)}} + h_2(t)v_2(t)]dt = \int_0^\omega [\frac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)} + e^{y_1(t-\tau_1)}}]dt \leq \bar{f}_1\omega, \quad (8)$$

$$\int_0^\omega [a_3(t) + h_3(t)v_3(t)]dt = \int_0^\omega [\frac{f_2(t)e^{y_2(t-\tau_2)}}{m_2(t)e^{y_3(t-\tau_2)} + e^{y_2(t-\tau_2)}}]dt \leq \bar{f}_2\omega, \quad (9)$$

$$\int_0^\omega \alpha_i(t)v_i(t)dt = \int_0^\omega [\beta_i(t)e^{y_i(t)}]dt. \quad (10)$$

By (7)-(10), we have

$$\begin{aligned} \int_0^\omega |\dot{y}_1(t)|dt < & \int_0^\omega |a_1(t)|dt + \int_0^\omega |b_1(t)e^{y_1(t)} + \tfrac{c_1(t)e^{y_2(t)}}{m_1(t)e^{y_2(t)}+e^{y_1(t)}} + h_1(t)v_1(t)|dt \\ = & \ 2\bar{a}_1\omega. \end{aligned} \quad (11)$$

$$\begin{aligned} \int_0^\omega |\dot{y}_2(t)|dt < & \int_0^\omega |a_2(t) + \tfrac{c_2(t)e^{y_3(t)}}{m_2(t)e^{y_3(t)}+e^{y_2(t)}} + h_2(t)v_2(t)|dt + \int_0^\omega |\tfrac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)}+e^{y_1(t-\tau_1)}}|dt \\ \leq & \ 2\bar{f}_1\omega. \end{aligned} \quad (12)$$

$$\int_0^\omega |\dot{y}_3(t)|dt < \int_0^\omega |a_3(t) + h_3(t)v_3(t)|dt + \int_0^\omega |\frac{f_2(t)e^{y_2(t-\tau_2)}}{m_2(t)e^{y_3(t-\tau_2)} + e^{y_2(t-\tau_2)}}|dt \leq 2\bar{f}_2\omega. \quad (13)$$

For $(y^T, v^T)^T \in Y$, there exists $\xi_i, \varsigma_i, \hat{\xi}_i, \hat{\varsigma}_i \in [0, \omega], i = 1, 2, 3,$ such that

$$y_i(\xi_i^-) = \inf_{t\in[0,\omega]} y_i(t), \qquad y_i(\varsigma_i^+) = \sup_{t\in[0,\omega]} y_i(t),$$

$$v_i(\hat{\xi}_i^-) = \inf_{t\in[0,\omega]} v_i(t), \qquad v_i(\hat{\varsigma}_i^+) = \sup_{t\in[0,\omega]} v_i(t).$$

Then from (7) and (11) we have

$$\bar{a}_1\omega \geq \int_0^\omega [b_1(t)e^{y_1(\xi_1^-)}]dt = \bar{b}_1\omega e^{y_1(\xi_1^-)}, \quad y_1(\xi_1^-) \leq \ln[\frac{\bar{a}_1}{\bar{b}_1}],$$

Then,

$$y_1(t) \leq \overline{\Delta}_1\omega + y_1(\xi_1^-) + \int_0^\omega |\dot{y}_1(t)|dt \leq \ln[\frac{\bar{a}_1}{\bar{b}_1}] + [\overline{\Delta}_1 + 2\bar{a}_1]\omega \triangleq L_1.$$

By (8)(12), we have

$$\begin{aligned} \bar{a}_2\omega & \leq \int_0^\omega [\tfrac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)}}]dt = \int_{-\tau_1}^{\omega-\tau_1} [\tfrac{f_1(s+\tau_1)e^{y_1(s)}}{m_1(t)e^{y_2(s)}}]ds \\ & \leq \int_{-\tau_1}^{\omega-\tau_1} [\tfrac{f_1(s+\tau_1)e^{y_1(\xi_2^-)}}{m_1(t)e^{y_2(\xi_2^-)}}]ds \leq \tfrac{e^{L_1}\bar{f}_1\omega}{m_1^l e^{y_2(\xi_2^-)}}. \end{aligned}$$

$$e^{y_2(\xi_2^-)} \leq \frac{\bar{f}_1}{m_1^l\bar{a}_2}e^{L_1} \leq \frac{\bar{f}_1\bar{a}_1}{m_1^l\bar{a}_2\bar{b}_1}e^{\omega(2\bar{a}_1+\overline{\Delta}_1)}, \quad y_2(\xi_2^-) \leq \ln[\frac{\bar{f}_1\bar{a}_1}{m_1^l\bar{a}_2\bar{b}_1}e^{\omega(2\bar{a}_1+\overline{\Delta}_1)}].$$

Therefore

$$y_2(t) \leq \overline{\Delta}_2\omega + y_2(\xi_2^-) + \int_0^\omega |\dot{y}_2(t)|dt \leq \ln[\frac{\bar{f}_1\bar{a}_1}{m_1^l\bar{a}_2\bar{b}_1}e^{\omega(2\bar{a}_1+\overline{\Delta}_1)}] + \omega(\overline{\Delta}_2 + 2\bar{f}_1) \triangleq L_2.$$

Similarly, it follows from (9) and (13) that

$$e^{y_3(\xi_3^-)} \leq \frac{\bar{f}_2}{m_2^l\bar{a}_3}e^{L_2},$$

$$y_3(t) \leq \overline{\Delta}_3\omega + y_3(\xi_3^-) + \int_0^\omega |\dot{y}_3(t)|dt \leq \ln[\frac{\bar{f}_1\bar{f}_2\bar{a}_1}{\bar{b}_1\bar{a}_2\bar{a}_3m_1^lm_2^l}e^{(2\bar{a}_1+2\bar{f}_1+\overline{\Delta}_1+\overline{\Delta}_2)\omega}] + \omega(\overline{\Delta}_3 + 2\bar{f}_2) \triangleq L_3.$$

By (10), we have

$$\int_0^{\omega} v_i(t)dt \le \frac{e^{y_i(\varsigma_i^+)}}{\alpha_i^l}\int_0^{\omega}\beta_i(t)dt \le \frac{e^{y_i(\varsigma_i^+)}}{\alpha_i^l}\omega\bar{\beta}_i,$$
$$\int_0^{\omega}\alpha_i(t)v_i(t)dt \le e^{L_i}\int_0^{\omega}\beta_i(t)dt \le e^{L_i}\omega\bar{\beta}_i, \tag{14}$$

It follows from (14) that

$$v_i(\hat{\xi}_i) \le e^{L_i}\frac{\bar{\beta}_i}{\bar{\alpha}_i}, \qquad \int_0^{\omega}|\dot{v}_i(t)|dt \le 2\int_0^{\omega}\alpha_i(t)v_i(t)dt \le 2e^{L_i}\omega\bar{\beta}_i,$$

Then,

$$v_i(t) \le v_i(\hat{\xi}_i) + \int_0^{\omega}|\dot{v}_i(t)|dt \le e^{L_i}[\frac{\bar{\beta}_i}{\bar{\alpha}_i} + 2\omega\bar{\beta}_i] \triangleq \hat{L}_i.$$

On the other hand,

$$\bar{a}_1\omega \le \int_0^{\omega}[b_1(t)e^{y_1(\varsigma_1^+)} + \frac{c_1(t)}{m_1(t)} + h_1(t)\hat{L}_1]dt = \bar{b}_1\omega e^{y_1(\varsigma_1^+)} + \frac{\bar{c}_1\omega}{m_1^l} + \bar{h}_1\hat{L}_1\omega.$$

$$y_1(\varsigma_1^+) \ge \ln[\frac{\bar{a}_1 - \frac{\bar{c}_1}{m_1^l} - \bar{h}_1\hat{L}_1}{\bar{b}_1}].$$

By (11),we have

$$y_1(t) \ge y_1(\varsigma_1^+) - \int_0^{\omega}|\dot{x}_1(t)|dt \ge \ln[\frac{\bar{a}_1 - \frac{\bar{c}_1}{m_1^l} - \bar{h}_1\hat{L}_1}{\bar{b}_1}] - 2\bar{a}_1\omega \triangleq l_1.$$

From (8) we have

$$\begin{aligned}\frac{\bar{f}_1\omega e^{y_1(\varsigma_2^+)}}{m_1^u e^{y_2(\varsigma_2^+)} + e^{y_1(\varsigma_2^+)}} &\le \int_{-\tau_1}^{\omega-\tau_1}\frac{f_1(s+\tau_1)e^{y_1(s)}}{m_1(s)e^{y_2(s)} + e^{y_1(s)}}ds \\ &= \int_0^{\omega}\frac{f_1(t)e^{y_1(t-\tau_1)}}{m_1(t)e^{y_2(t-\tau_1)} + e^{y_1(t-\tau_1)}}dt \le \bar{a}_2\omega + \frac{\bar{c}_2\omega}{m_2^l} + \bar{h}_2\hat{L}_2\omega,\end{aligned}$$

$$e^{y_2(\varsigma_2^+)} \ge \frac{\bar{f}_1 - \bar{a}_2 - \frac{\bar{c}_2}{m_2^l} - \bar{h}_2\hat{L}_2}{m_1^u(\bar{a}_2 + \frac{\bar{c}_2}{m_2^l} + \bar{h}_2\hat{L}_2)}e^{y_1(\varsigma_2^+)} \ge \frac{\bar{f}_1 - \bar{a}_2 - \frac{\bar{c}_2}{m_2^l} - \bar{h}_2\hat{L}_2}{m_1^u(\bar{a}_2 + \frac{\bar{c}_2}{m_2^l} + \bar{h}_2\hat{L}_2)}e^{l_1},$$

$$y_2(\varsigma_2^+) \ge \ln[\frac{\bar{f}_1 - \bar{a}_2 - \frac{\bar{c}_2}{m_2^l} - \bar{h}_2\hat{L}_2}{m_1^u(\bar{a}_2 + \frac{\bar{c}_2}{m_2^l} + \bar{h}_2\hat{L}_2)}e^{l_1}],$$

$$y_2(t) \ge y_2(\varsigma_2^+) - \int_0^{\omega}|\dot{y}_2(t)|dt \ge \ln[\frac{\bar{f}_1 - \bar{a}_2 - \frac{\bar{c}_2}{m_2^l} - \bar{h}_2\hat{L}_2}{m_1^u(\bar{a}_2 + \frac{\bar{c}_2}{m_2^l} + \bar{h}_2\hat{L}_2)}e^{l_1}] - 2\bar{f}_1\omega \triangleq l_2.$$

Similarly, it follows from (9) that

$$y_3(\varsigma_3^+) \ge \ln[\frac{\bar{f}_2 - \bar{a}_3 - \bar{h}_3\hat{L}_3}{m_2^u(\bar{a}_3 + \bar{h}_3\hat{L}_3)}e^{l_2}],$$

$$y_3(t) \ge y_3(\varsigma_3^+) - \int_0^{\omega}|\dot{y}_3(t)|dt \ge \ln[\frac{\bar{f}_2 - \bar{a}_3 - \bar{h}_3\hat{L}_3}{m_2^u(\bar{a}_3 + \bar{h}_3\hat{L}_3)}e^{l_2}] - 2\bar{f}_2\omega \triangleq l_3.$$

By (10), we have

$$v_i(\hat{\varsigma})\omega\bar{\alpha}_i \ge \int_0^{\omega}\alpha_i(t)v_i(t)dt \ge e^{l_i}\omega\bar{\beta}_i.$$

$$v_i(t) \geq v_i(\varsigma_i) - \int_0^\omega |\dot{v}_i(t)|dt \geq \frac{e^{l_i}\bar{\beta}_i}{\bar{\alpha}_i} - 2\omega\bar{\beta}_i \triangleq \hat{l}_i.$$

Then, for $\lambda \in (0,1)$,

$$l_i \leq y_i(t) \leq L_i, \hat{l}_i \leq v_i(t) \leq \hat{L}_i. \tag{15}$$

Clearly , $l_i, L_i, \hat{l}_i, \hat{L}_i, i = 1,2,3$ are independent of λ .

We take $\Omega = \{U \in Y | \|U\| < H\}$, here H is taken sufficiently large such that $H > \max\limits_{1\leq i\leq 3}\{|l_i| + |L_i| + |\hat{l}_i| + |\hat{L}_i|\}$. Now we check the conditions of Lemma 1. By (15),one can conclude that for each $\lambda \in (0,1)$, $y \in \partial\Omega$, $LU \neq \lambda NU$.

considering to $U = (y_1, y_2, y_3, v_1, v_2, v_3)^T \in R^6$ of the system of algebraic equations

$$\begin{cases} \bar{a}_1 - \bar{b}_1 e^{y_1} - \mu(\frac{\bar{c}_1 e^{y_2}}{\bar{m}_1 e^{y_2} + e^{y_1}} + \bar{h}_1 v_1) = 0, \\ -\bar{a}_2 + \frac{\bar{f}_1 e^{y_1}}{\bar{m}_1 e^{y_2} + e^{y_1}} - \mu(\frac{\bar{c}_2 e^{y_3}}{\bar{m}_2 e^{y_3} + e^{y_2}} + \bar{h}_2 v_2) = 0, \\ -\bar{a}_3 + \frac{\bar{f}_2 e^{y_2}}{\bar{m}_2 e^{y_3} + e^{y_2}} - \mu\bar{h}_3 v_3 = 0, \\ \bar{\alpha}_i v_i - \bar{\beta}_i e^{y_i} = 0, \qquad\qquad i = 1,2,3. \end{cases} \tag{16}$$

here $\mu \in [0,1]$. For any $\mu \in [0,1]$, the solution $(y^T, v^T)^T$ of algebraic equations (16) satisfies

$$l_i \leq y_i \leq L_i, \hat{l}_i \leq v_i \leq \hat{L}_i. \tag{17}$$

For any $U \in \partial\Omega \bigcap \ker L, U$ is a constant vector in R^6 with $\|U\| = H$, we have

$$QNU = \left(\begin{pmatrix} \bar{a}_1 - \bar{b}_1 e^{y_1} - \frac{\bar{c}_1 e^{y_2}}{\bar{m}_1 e^{y_2} + e^{y_1}} - \bar{h}_1 v_1 \\ -\bar{a}_2 + \frac{\bar{f}_1 e^{y_1}}{\bar{m}_1 e^{y_2} + e^{y_1}} - \frac{\bar{c}_2 e^{y_3}}{\bar{m}_2 e^{y_3} + e^{y_2}} - \bar{h}_2 v_2 \\ -\bar{a}_3 + \frac{\bar{f}_2 e^{y_2}}{\bar{m}_2 e^{y_3} + e^{y_2}} - \bar{h}_3 v_3, \\ (\bar{\alpha}_i v_i - \bar{\beta}_i e^{y_i})_{3\times 1} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \right)$$ By(17), for any $U \in \partial\Omega \bigcap \ker L$, we have $QNU \neq 0$.

In order to calculate Brouwer Degree, we need make a homotopy mapping.

$$G(\mu, U) = \mu QNU + (1-\mu)H(U), \qquad \mu \in [0,1],$$
$$H(U) = \begin{pmatrix} \bar{a}_1 - \bar{b}_1 e^{y_1} \\ -\bar{a}_2 + \frac{\bar{f}_1 e^{y_1}}{\bar{m}_1 e^{y_2} + e^{y_1}} \\ -\bar{a}_3 + \frac{\bar{f}_2 e^{y_2}}{\bar{m}_2 e^{y_3} + e^{y_2}}, \\ (\bar{\alpha}_i v_i - \bar{\beta}_i e^{y_i})_{3\times 1} \end{pmatrix}, \qquad U = (y^T, v^T)^T.$$

From (17), for any $U \in \partial\Omega \bigcap \ker Land\mu \in [0,1]$, we know $G(\mu, U) \neq 0$. Because of $\mathrm{Im}Q = \ker L$, we take $J = I$, So, due to homotopy invariance theorem we obtain

$$\begin{aligned} \deg(JQN, \Omega \cap KerL, 0) &= \deg(\quad QN, \Omega \cap \ker L, 0) \\ &= \deg(\quad H, \Omega \cap \ker L, 0) \\ &= \deg(\quad \bar{a}_1 - \bar{b}_1 e^{y_1}, -\bar{a}_2 + \tfrac{\bar{f}_1 e^{y_1}}{\bar{m}_1 e^{y_2} + e^{y_1}}, -\bar{a}_3 + \tfrac{\bar{f}_2 e^{y_2}}{\bar{m}_2 e^{y_3} + e^{y_2}}, \\ &\qquad\quad \bar{\alpha}_1 v_1 - \bar{\beta}_1 e^{y_1}, \bar{\alpha}_2 v_2 - \bar{\beta}_2 e^{y_2}, \bar{\alpha}_3 v_3 - \bar{\beta}_3 e^{y_3}, \Omega \cap KerL, 0), \end{aligned}$$

and because the following algebraic equations has a unique solution,

$$\begin{cases} \bar{a}_1 - \bar{b}_1 z_1 = 0, \\ -\bar{a}_2 + \frac{\bar{f}_1 z_1}{\bar{m}_1 z_2 + z_1} = 0, \\ -\bar{a}_3 + \frac{\bar{f}_2 z_2}{\bar{m}_2 z_3 + z_2} = 0, \\ \bar{\alpha}_i u_i - \bar{\beta}_i z_i = 0, i = 1,2,3. \end{cases}$$
$$z_1^* = \frac{\bar{a}_1}{\bar{b}_1}, \quad z_2^* = \frac{(\bar{f}_1 - \bar{a}_2)\bar{a}_1}{\bar{m}_1 \bar{a}_2 \bar{b}_1}, \quad z_3^* = \frac{(\bar{f}_2 - \bar{a}_3)(\bar{f}_1 - \bar{a}_2)\bar{a}_1}{\bar{m}_1 \bar{m}_2 \bar{a}_2 \bar{a}_3 \bar{b}_1}, \quad u_i^* = \frac{\bar{\beta}_i z_i^*}{\bar{\alpha}_i}.$$

Then there is not difficult to know that $H(U)=0$ has also a unique solution, we obtain

$$\begin{aligned}\deg(H,\Omega\cap\ker L,0)= &\ \operatorname{sign}\begin{vmatrix} -\bar{b}_1 & 0 & 0 & 0 & 0 & 0\\ \frac{\bar{f}_1\bar{m}_1 z_2^*}{(\bar{m}_1 z_2^*+z_1^*)^2} & \frac{-\bar{f}_1\bar{m}_1 z_1^*}{(\bar{m}_1 z_2^*+z_1^*)^2} & 0 & 0 & 0 & 0\\ 0 & \frac{\bar{f}_2\bar{m}_2 z_3^*}{(\bar{m}_2 z_3^*+z_2^*)^2} & \frac{-\bar{f}_2 m_2 y_2^*}{(\bar{m}_2 z_3^*+z_2^*)^2} & 0 & 0 & 0\\ -\bar{\beta}_1 & 0 & 0 & \bar{\alpha}_1 & 0 & 0\\ 0 & -\bar{\beta}_2 & 0 & 0 & \bar{\alpha}_2 & 0\\ 0 & 0 & -\bar{\beta}_3 & 0 & 0 & \bar{\alpha}_3\end{vmatrix}\\ = &\ \operatorname{sign}\left|-\frac{\bar{\alpha}_1\bar{\alpha}_2\bar{\alpha}_3\bar{b}_1\bar{f}_1\bar{f}_2\bar{m}_1\bar{m}_2 z_1^* z_2^*}{(\bar{m}_1 z_2^*+z_1^*)^2(\bar{m}_2 z_3^*+z_2^*)^2}\right| = -1\neq 0.\end{aligned}$$

By now we have proved that Ω satisfies all the requirements in Lemma 1. Hence, (5) has at least one $\omega-$periodic solution $((y^*(t))^T,(v^*(t))^T)^T$. Accordingly, system (4) has at least one $\omega-$periodic solution $((x^*(t))^T,(v^*(t))^T)^T=((e^{y^*(t)})^T,(v^*(t))^T)^T$ with strictly positive components. This completes the proof.

3.1 Global asymptotic stability of periodic solutions

Theorem 2 In addition to the condition of Theorem 1, assume further that

$$b^l>\frac{f_1^m}{m_1^l l_2+l_1}+\beta_1^u,\quad \frac{c_1^l}{m_1^u L_2+L_1}>\frac{f_2^m}{m_2^l l_3+l_2}+\beta_2^u,\quad \frac{c_2^l}{m_2^u L_3 L_2}>\beta_3^u,\quad h_i^l+\alpha_i^l>0$$

Then system (4) has a unique positive $\omega-$periodic solution which is globally asymptotically stable.

Proof. Based on the conclusion of Theorem 1, we need only to verify the globally asymptotically stability of positive periodic solutions of (4). Let $((x^*(t))^T,(v^*(t))^T)^T$ be a positive $\omega-$periodic solution of system (4). and $((x(t))^T,(v(t))^T)^T$ be any positive solution of system (4). We define a Lyapunov function

$$\begin{aligned}V(t)= &\ \sum_{i=1}^{3}[|\ln x_i(t)-\ln x_i^*(t)|+|v_i(t)-v_i^*(t)|]\\ &+\int_{t-\tau_1}^{t}\frac{f_1(t)}{m_1^l l_2+l_1}|x_1(u)-x_1^*(u)|du+\int_{t-\tau_2}^{t}\frac{f_2(t)}{m_2^l l_3+l_2}|x_2(u)-x_2^*(u)|du.\end{aligned}$$

When $t=t_k, k\in N$,

$$V(t_k)-V(t_k^-)=\sum_{i=1}^{3}[|\ln(1+b_{ik})x_i(t_k^-)-\ln(1+b_{ik})x_i^*(t_k^-)|-|\ln x_i(t_k^-)-\ln x_i^*(t_k^-)|]=0,$$

then $V(t)$ is a a continuous function.

When $t\neq t_k, k\in N$, calculating the upper right derivative of $V(t)$ along solutions of system (4),we derive

$$\begin{aligned}D^+V(t)\leq &\ [-b^l+\frac{f_1^m}{m_1^l l_2+l_1}+\beta_1^u]|x_1(u)-x_1^*(u)|+[-\frac{c_1^l}{m_1^u L_2+L_1}+\frac{f_2^m}{m_2^l l_3+l_2}+\beta_2^u]|x_2(u)-x_2^*(u)|\\ &+[-\frac{c_2^l}{m_2^u L_3+L_2}+\beta_3^u]|x_3(u)-x_3^*(u)|+\sum_{i=1}^{3}[(-h_i^l-\alpha_i^l)|v_i(t)-v_i^*(t)|],\end{aligned}$$

By assumption conditions of theorem 2, there exists a constant

$$\gamma=\min\{b^l-\frac{f_1^m}{m_1^l l_2+l_1}-\beta_1^u,\frac{c_1^l}{m_1^u L_2+L_1}-\frac{f_2^m}{m_2^l l_3+l_2}-\beta_2^u,\frac{c_2^l}{m_2^u L_3 L_2}-\beta_3^u,h_i^l+\alpha_i^l\}>0,$$

such that

$$D^+V(t)\leq-\gamma\sum_{i=1}^{3}[|x_i(t)-x_i^*(t)|+|v_i(t)-v_i^*(t)|]. \tag{18}$$

Then

$$\gamma\sum_{i=1}^{3}\int_0^t[|x_i(t)-x_i^*(t)|+|v_i(t)-v_i^*(t)|]dt \le V(0)-V(t), \quad t\ge 0,$$

$$\int_0^{\infty}[|x_i(t)-x_i^*(t)|dt<\infty, \qquad \int_0^{\infty}[|v_i(t)-v_i^*(t)|dt<\infty, \qquad i=1,2,3.$$

Therefore, $|x_i(t)-x_i^*(t)|$ and $|v_i(t)-v_i^*(t)|$ are bounded on $[0,\infty)$, and it is easy to see that their derivative are also bounded. By Barbalat's lemma(lemma 2), we conclude that

$$\lim_{t\to\infty}|x_i(t)-x_i^*(t)|=0, \qquad \lim_{t\to\infty}|v_i(t)-v_i^*(t)|=0, \qquad i=1,2,3. \tag{19}$$

Further, there exists a positive number M such that $|\ln x_i(t)-\ln x_i^*(t)| \ge \frac{|x_i(t)-x_i^*(t)|}{M}$, Thus,

$$V(t)\ge \frac{1}{M}\sum_{i=1}^{3}[|x_i(t)-x_i^*(t)|+|v_i(t)-v_i^*(t)|].$$

Combining (18) and (19), we have completed that system (4) has a unique positive $\omega-$periodic solution which is globally asymptotically stable. This completes the proof.

Note: The results in this paper can be extended to a n species food chain model with ratio-dependent functional response and feedback controls.

4 CONCLUSIONS

This paper has considered a food chain system with ratio-dependent functional response, pulse, delays and feedback controls. The sufficient conditions of the existence and global asymptotic stability of positive periodic solution are derived. The article provides a good theoretical basis for the further study for the food chain system with ratio-dependent functional response and feedback controls.

References

Cheng, R.F., Li, H. (2007). The bifurcation and limit cycle of predator-prey gaining model with Holling III function response. Jornal of Northeast Normal University, 39(1), 17-21.

Chen, L.S., et al. (2003) Mathematical ecological model and methods, Chengdu:Sichuan Science Press,

Gopalsamy, K ., Weng, P.X. (1993). Feeback Regulation of logisticgrowth. Internet J Math and Math Sci, 16, 177-192

Gopalsamy, K, (1992). Stability and oscillations in delay differential equtions of population dynamics. Kluwer Academic, Dordrecht/Norwell,MA.

Gaines, R E ., Mawhin, J L. (1997) Coincidence degree and nonlinear differential equations.Berlin:Springer.

Li, H., Wang, Y.F. (2009). Existence of positive periodic solution of food chain system with feedback controls and ratio-dependent functional response. Journal of Jilin University (Science Edition), 47(2), 221-227.

Li, L., et al. (2003), The stability and persistence on a environment mathematical model. Acta Mathematical Applicatae Sinica, 26(1), 149-157.

Si, R.X., Chen, S.Y. (2007).The persistence for nonautonomous three species predator-prey system with feedback control. Journal of Henan Normal University (Natural Science), 35(1), 5-8.

Xu, R,. Chen, L.S (2001). Persistence and global stability for three-species ratio-dependent predator-prey system with time delays. Sys.Sci.and. Math.Scis, 21(2), 204-212

Yang, Z.C., Xu, D.Y. (2009). Periodic solutions and stability of impulsive type competition system with infinitely distributed delays and feedback controls. Acta Mathematicae Applicatae Sinica, 32(1), 132-142.

Zhang, S.W. et al.(2005). A food chain model with impulsive perturbations and Holling IV functional response. Chaos,Solitons and Fractals, 26(4), 855-866.
