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Solitary Wave, Periodic Cusp Wave and Compactons of the (2+1)-Dimensional KP-Like K(m,n) Equation

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Research Article

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Abstract

By using the bifurcation theory of dynamical systems to the generalized KP equation, under different parametric conditions, various sufficient conditions to guarantee the existence of the solitary wave solutions, periodic cusp wave solutions and compactons solutions are given. Some exact explicit parametric representations of the above waves are determined.

Keywords: Solitary wave, periodic cusp wave, compactons, (2+1)-dimensional KP-like K(m,n) equation.

1 Introduction

Kadomtsev and Petviashvili [1] first considered the following KP equation:

$$u_{xt}+\sim\left(u_x^2+uu_{xx}\right)+¥u_{xxxx}+£u_{yy}=0, \quad (1.1)$$

where $\sim, ¥, £$ are arbitrary constants. This equation is used to describe weakly dispersive and nonlinear medium perturbation. The KP equation can also be regarded as KdV equation in the promotion of two-dimensional space. KP equation in (2+1)-dimensional equation occupies a very important position which has N-soliton solutions, infite symmetries and conservation laws, Painlev properties and so on [2-5].

In this paper, we shall study all solitary waves [6], compactons [7] and periodic cusp waves in the parameter space of the following (2+1)- dimensional KP-like K(m,n) equation:

$$a\left(u^m\right)_{xt}+\sim\left(u_x{}^2+uu_{xx}\right)+¥\left(u^n\right)_{xxxx}+£\left(u^n\right)_{yy}=0, \quad (1.2)$$

where $\sim, ¥, £$ are arbitrary constants, m, n and a are non-zero integers and a non-zero arbitrary constant, respectively. This equation is to promote the (2+1)-dimensional KP equation which has similar K (m, n) equation form [8-10]. I believe in the future, as the equation continues to be studied, it will be like to other equations used in many physics areas such as nonlinear optics, plasmas, fluid mechanics, condensed matter and many more. Specially, when $a=m=n=1$, Eq. (1.2) becomes

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KP equation. But in this paper we consider

$m > n \geq 3, ac \neq 0$ and we shall study all traveling wave solutions in the parameter space of this system. Let $u(x, y, t) = ¤(x + wy - ct) = ¤(¶)$, where c and w are the wave speed and the wave number on the y-direction, respectively. Then Eq.(1.2) becomes

$-ac\left(¤^m\right)'' + \sim\left(\left(¤'\right)^2 + ¤¤''\right) + ¥\left(¤^n\right)'''' + £w^2\left(¤^n\right)'' = 0$, (1.3) where"″" is the derivative with respect to ¶. Integrating (1.3) twice, setting the constants of integration to be zero we have the following ordinary differential equation

$$-ac¤^m + \frac{1}{2}\sim ¤^2 + ¥\left(¤^n\right)'' + £w^2¤^n = 0. \qquad (1.4)$$

Let $q = -\frac{\sim}{2ac}, p = -\frac{w^2£}{ac}, r = -\frac{¥}{ac}$ .Then Eq.(1.4) is equivalent to the following two-dimensional system:

$$\frac{d¤}{d¶} = z, \quad \frac{dz}{d¶} = -\frac{q¤^2 + p¤^n + ¤^m + rn(n-1)¤^{n-2}z^2}{rn¤^{n-1}}. \qquad (1.5)$$

With the first integral

$$H(¤, z) = \frac{1}{2}rn¤^{2(n-1)}z^2 + ¤^{n+2}\left[\frac{q}{n+2} + \frac{p}{2n}¤^{n-2} + \frac{1}{m+n}¤^{m-2}\right] = h. \qquad (1.6)$$

System (1.5) is a 5-parameter planar dynamical system depending on the parameter group (m, n, p, q, r). For different m, n and a fixed r, we shall investigate the bifurcations of phase portraits of (1.5) in the phase plane $(¤, z)$ as the parameters p and q are vary. The bifurcation theory of dynamical systems plays an important role in our study [11].

Clearly, the right hand of the second equation in (1.5) is not continuous when $¤ = 0$. In other words, on the above straight line of the phase plane $(¤, z)$, $¤_¶''$ has no definition. This implies that the smooth system (1.2) sometimes has non-smooth traveling wave solutions. This phenomenon has been studied by some authors [12-16].

2 Bifurcations of Phase Portraits of (1.5)

In this section, we study all possible periodic annuli, homoclinic and heteroclinic orbits defined by the vector fields of (1.5) depending on the parameter space (m, n, p, q, r). System(1.5) has the same topological phase portraits as the following system

$$\frac{d¤}{d¸} = rn¤^{n-1}z, \quad \frac{dz}{d¸} = -\left[q¤^2 + p¤^n + ¤^m + rn(n-1)¤^{n-2}z^2\right], \qquad (2.1)$$

except for the straight lines $¤ = 0$, where $d¶ = rn¤^{n-1}d¸$. Now, the straight lines $¤ = 0$ is an integral invariant straight line of (2.1). Denote that

$$f\left(¤\right)=q+p¤^{n-2}+¤^{m-2},\quad f'\left(¤\right)=\left(m-2\right)¤^{n-3}\left[¤^{m-n}+\frac{p\left(n-2\right)}{m-2}\right].\text{(2.2)}$$

For $m-n=2l(l\in Z^{+}),m-2=2m_1-1,n-2=2n_1-1,m_1,n_1\in Z^{+},p<0$,when $¤=¤_0=\left[-\frac{p\left(n-2\right)}{m-2}\right]^{\frac{1}{m-n}},f'\left(\pm¤_0\right)=0$.

We have $f(\pm¤_0)=q\pm\left[-\frac{p(n-2)}{m-2}\right]^{\frac{m-2}{m-n}}\left(\frac{n-m}{n-2}\right)$,which imply respectively the relations in the $\left(p,q\right)$-parameter plane

$$L_a:\quad q=\left(\frac{m-n}{n-2}\right)\left[-\frac{p\left(n-2\right)}{m-2}\right]^{\frac{m-2}{m-n}},\quad L_b:\quad q=-\left(\frac{m-n}{n-2}\right)\left[-\frac{p\left(n-2\right)}{m-2}\right]^{\frac{m-2}{m-n}}.$$

For $m-n=2l(l\in Z^{+}),m-2=2m_1,n-2=2n_1,p<0$, when $¤=¤_0=\left[-\frac{p(n-2)}{m-2}\right]^{\frac{1}{m-n}}$, $f'(\pm¤_0)=0$ .We have $f(\pm¤_0)=q+\left[-\frac{p(n-2)}{m-2}\right]^{\frac{m-2}{m-n}}\left(\frac{n-m}{n-2}\right)$,which imply respectively the relations in the $\left(p,q\right)$-parameter plane

$$L_a:\quad q=\left(\frac{m-n}{n-2}\right)\left[-\frac{p\left(n-2\right)}{m-2}\right]^{\frac{m-2}{m-n}}.$$

For $m-n=2l-1(l\in Z^{+}),m-2=2m_1,n-2=2n_1-1$,when $¤=¤_0=\left[-\frac{p(n-2)}{m-2}\right]^{\frac{1}{m-n}}$, $f'(¤_0)=0$. We have $f(¤_0)=q+\left[-\frac{p(n-2)}{m-2}\right]^{\frac{m-2}{m-n}}\left(\frac{n-m}{n-2}\right)$,which imply respectively the relations in the $\left(p,q\right)$-parameter plane

$$L_c:\quad q=\left(\frac{m-n}{n-2}\right)\left[\frac{p\left(n-2\right)}{m-2}\right]^{\frac{m-2}{m-n}}.$$

For $m-n=2l-1(l\in Z^{+}),m-2=2m_1,n-2=2n_1-1$, when $¤=¤_0=\left[-\frac{p(n-2)}{m-2}\right]^{\frac{1}{m-n}}$,

$f'(¤_0) = 0$. We have $f(¤_0) = q + \left[-\frac{p(n-2)}{m-2}\right]^{\frac{m-2}{m-n}} \left(\frac{n-m}{n-2}\right)$,which imply respectively the relations in the (p,q)-parameter plane

$$L_d: \quad q = -\left(\frac{m-n}{n-2}\right)\left[\frac{p(n-2)}{m-2}\right]^{\frac{m-2}{m-n}}.$$

Let $M(¤_e, z_e)$ be the coefficient matrix of the linearized system of (2.1) at an equilibrium point $(¤_e, z_e)$. Then, we have

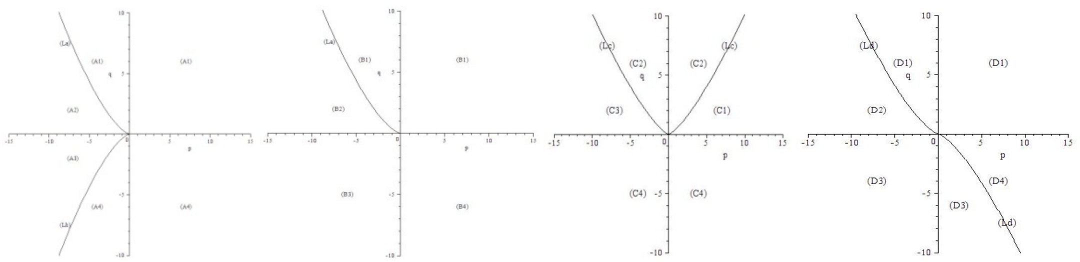
$$J(¤_i, 0) = \det\left(M(¤_i, 0)\right) = rn¤_i^{\,n}\left(2q + pn¤_i^{\,n-2} + m¤_i^{\,m-2}\right).$$

If $J < 0$ then the equilibrium point is a saddle point; if $J > 0$ and $Trace\left(M(¤_e, z_e)\right) = 0$, then it is a center point; if $J > 0$ and $\left(Trace\left(M(¤_e, z_e)\right)\right)^2 - 4J(¤_e, z_e) > 0$, then it is a node; if $J = 0$ and the index of the equilibrium point is 0 then it is a cusp; otherwise, it is a high order equilibrium point.

For the function defined by (1.6), we denote that

$$h_i = H(¤_i, 0) = ¤_i^{\,n+2}\left[\frac{q(m-2)}{(n+2)(m+n)} + \frac{p(m-n)}{2n(m+n)}¤_i^{\,n-2}\right], \quad i = 1—5.$$

We next use the above statements to consider the bifurcations of the phase portraits of (2.1). In the (p,q)-parameter plane, the curves partition it into 4 regions for $m-n=2l$ or $m-n=2l-1$ shown in Fig.1 (1-1),(1-2),(1-3), and (1-4), respectively.



(1-1) $m-n=2l$, (1-2) $m-n=2l$, (1-3) $m-n=2l-1$,(1-4) $m-n=2l-1$,

$n=2n_1+1$. $n=2n_1+2$. $n=2n_1+1$. $n=2n_1+2$.

Fig. 1 The bifurcation set of (1.5) in (p,q)-parameter plane, $m_1, n_1 \in Z^+$.

I. The case $q \neq 0$. We use Fig.2, Fig.3, Fig.4, and Fig.5 to show the bifurcations of the phase portraits of (2.1) which has solitary wave solutions, periodic cusp wave solutions and compactons solutions.

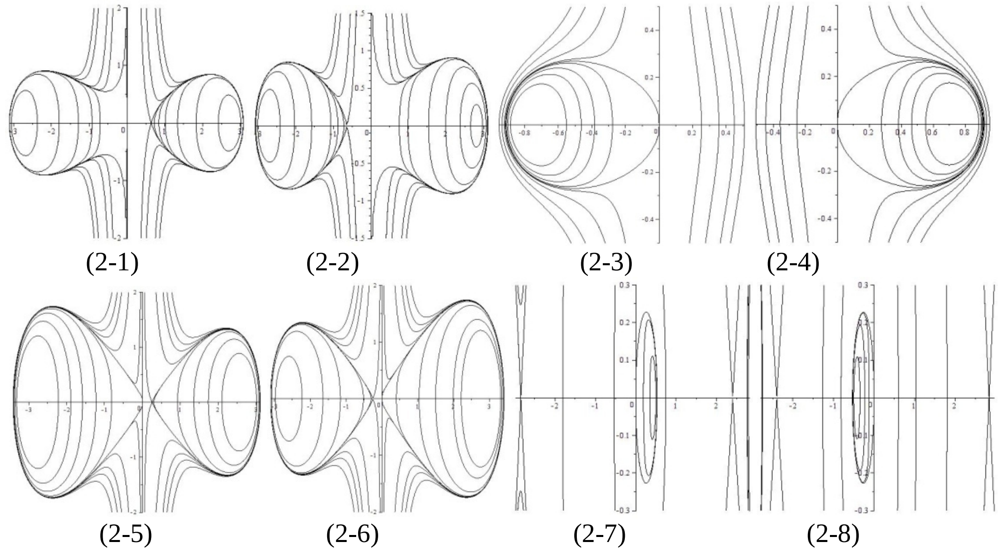


Fig. 2 The phase portraits of (1.5) for $m-n=2l, n=2n_1+1, l, n_1 \in Z^+$

(2-1) $r>0, n_1 \geq 2,\ (p,q) \in (A_2)$, (2-2) $r>0, n_1 \geq 2,\ (p,q) \in (A_3)$, (2-3) $r>0, n_1=1,$ $(p,q) \in (A_1)$, (2-4) $r>0, n_1=1,\ (p,q) \in (A_4)$, (2-5) $r>0, n_1=1,\ (p,q) \in (A_2)$, (2-6) $r>0, n_1=1,\ (p,q) \in (A_3)$,(2-7) $r<0, n_1=1,\ (p,q) \in (A_2)$, (2-8) $r<0, n_1=1,\ (p,q) \in (A_3)$

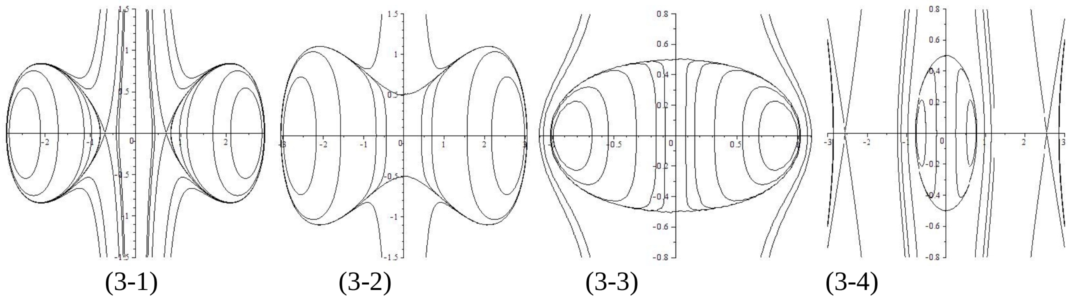
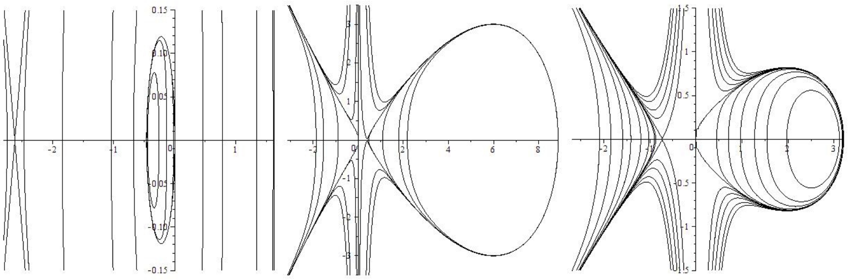
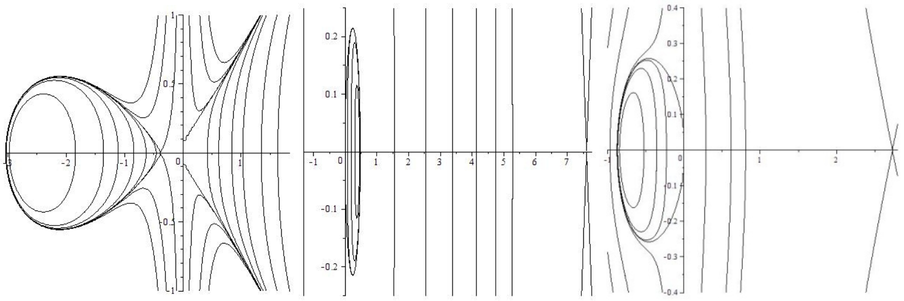


Fig. 3 The phase portraits of (1.5) for $m-n=2l, n=2n_1+2, l, n_1 \in Z^+$.

(3-1) $r>0, (p,q) \in (B_2)$, (3-2) $r>0, n_1=1, (p,q) \in (B_3)$,(3-3) $r>0, n_1=1, (p,q) \in (B_4)$, (3-4) $r<0, n_1=1, (p,q) \in (B_2)$.



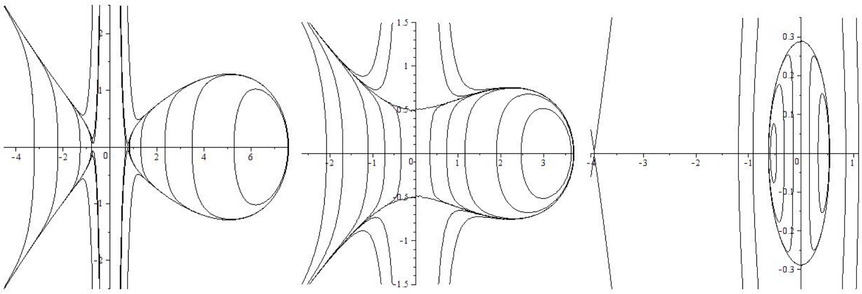
(4-1) (4-2) (4-3)



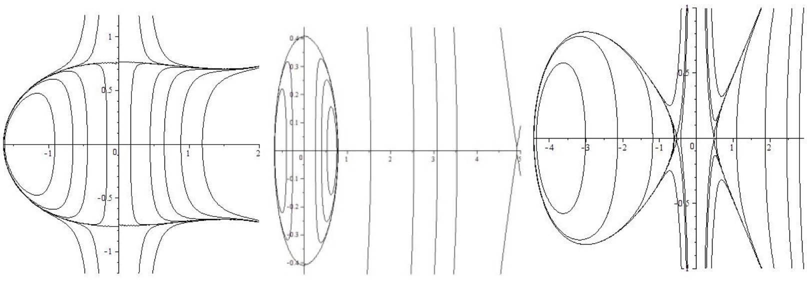
(4-4) (4-5) (4-6)

Fig. 4 The phase portraits of (1.5) for $m-n=2l-1, n=2n_1+1, l, n_1 \in Z^+$.

(4-1) $r>0, n_1=1, (p,q)\in(C_1)$, (4-2) $r>0, n_1=1, (p,q)\in(C_3)$, (4-3) $r>0, n_1=1,\ (p,q)\in(C_4)$, (4-4) $r<0, (p,q)\in(C_1)$, (4-5) $r<0, n_1=1, (p,q)\in(C_3)$, (4-6) $r<0, n_1=1,\ (p,q)\in(C_4)$.



(5-1) (5-2) (5-3)



(5-4) (5-5) (5-6)

Fig. 5 The phase portraits of (1.5) for $m-n=2l-1, n=2n_1+2, l, n_1 \in Z^+$.

(5-1) $r>0, (p,q)\in(D_2)$, (5-2) $r>0, n_1=1, (p,q)\in(D_3)$, (5-3) $r>0, n_1=1, (p,q)\in(D_4)$, (5-4) $r<0, n_1=1, (p,q)\in(D_1)$, (5-5) $r<0, n_1=1, (p,q)\in(D_2)$, (5-6) $r<0, (p,q)\in(D_4)$.

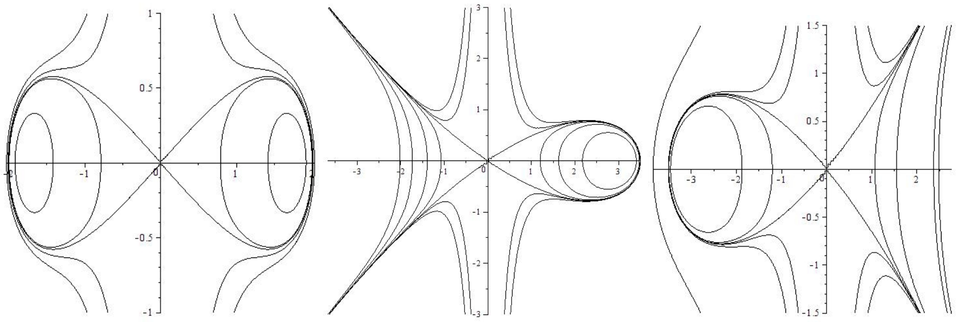
II. The case $q = 0$. We consider the system

$$\frac{d\phi}{d\xi} = rn\phi z, \quad \frac{dz}{d\xi} = -\left[p\phi^2 + \phi^{m-n+2} + rn(n-1)z^2 \right], \tag{2.3}$$

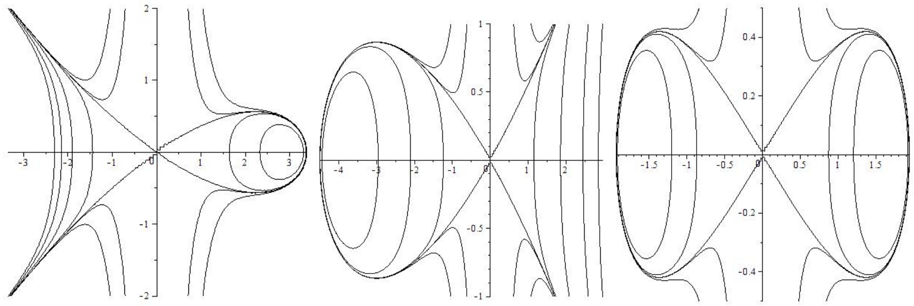
with the first integral

$$H(\phi, z) = \frac{1}{2} rn\phi^{2(n-1)} z^2 + \phi^{n+2}\left[\frac{p}{2n}\phi^{n-2} + \frac{1}{m+n}\phi^{m-2} \right] = h. \tag{2.4}$$

Fig.6 and Fig.7 show respectively the phase portraits of (2.3) which has solitary wave solutions, periodic cusp wave solutions and compactons solutions.



(6-1) (6-2) (6-3)



(7-1) (7-2) (7-3)

Fig. 6 for $q = 0, n = 2n_1 + 1, n_1 \in Z^+$, **Fig. 7 for** $q = 0, n = 2n_1 + 2, n_1 \in Z^+$.

(6-1) $r > 0, m = 2m_1 + 1, m_1 > n_1, p < 0$, (6-2) $r > 0, m = 2m_1 + 2, m_1 > n_1, p < 0$, (6-3) $r < 0$, $m = 2m_1 + 2, m_1 > n_1, p > 0$, (7-1) $r > 0, m = 2m_1 + 1, m_1 > n_1, p < 0$, (7-2) $r < 0, m = 2m_1 + 1$, $m_1 > n_1, p > 0$, (7-3) $r > 0, m = 2m_1 + 2, m_1 > n_1, p < 0$.

3 Exact Explicit Parametric Representations of Travelling Wave Solutions of(1.5)

In this section, we only give some exact explicit parametric representations of solitary wave solutions, periodic cusp wave solutions and compactons solutions. Because the phase portraits (6-2), (7-1), (2-3), (2-7), (3-2), (4-1), (4-2), (4-3), (5-1), (5-2) and (5-3), are there flections of the phase portraits (6-3), (7-2), (2-4), (2-8), (3-3), (4-5), (4-4), (4-6), (5-6),(5-4) and (5-5), with respect to the z-axis. So we consider the above phase portraits, the only discussion the phase portraits (6-2), (7-1), (2-3), (2-7), (3-2), (4-1), (4-2), (4-3), (5-1),(5-2) and (5-3).

(1) Suppose that $n = 2n_1 + 1, m = 2m_1 + 1, m_1 > n_1, m_1, n_1 \in Z^+, r > 0, p < 0, q = 0$ (see Fig. 6 (6-

1)). Notice that $H(0,0)=0=h_2$. We see from (2.4) that the arch curve connecting A(0,0). The arch curve has the algebraic equation

$$z^2=\frac{1}{r(2n_1+1)}\left[-\frac{p}{2n_1+1}¤^2-\frac{1}{m_1+n_1+1}¤^{2(m_1-n_1+1)}\right]. \tag{3.1}$$

Thus, by using the first equation of (1:5) and (3.1), we obtain the parametric representation of this arch as follows:

$$¤(¶)=\pm\left[\sqrt{\frac{-p(m_1+n_1+1)}{2n_1+1}}\,\mathrm{sech}\left(\frac{m_1-n_1}{2n_1+1}\sqrt{\frac{-p}{r}}¶\right)\right]^{\frac{1}{m_1-n_1}}. \tag{3.2}$$

So (3.2) gives rise to two solitary wave solutions of peak type and valley type of (1.2).

(2) Suppose that $n=2n_1+1, m=2n_1+2, n_1\in Z^+, rp<0, q=0$ (see Fig. 6 (6-2) or (6-3)). By using similar method of (1), we obtain the parametric representation of this arch as follows:

$$¤(¶)=\frac{-p(4n_1+3)}{4n_1+2}\left[1-\tanh^2\left(\frac{1}{4n_1+2}\sqrt{\frac{-p}{r}}¶\right)\right]. \tag{3.3}$$

So (3.3) gives rise to a solitary wave solutions of peak type and valley type of (1.2).

(3)Suppose that $n=2n_1+2, m=2n_1+3, n_1\in Z^+, rp<0, q=0$ (see Fig. 7 (7-1) or (7-2)). By using similar method of (1), we obtain the parametric representation of this arch as follows:

$$¤(¶)=\frac{-p(4n_1+5)}{4(n_1+1)}\left[1-\tanh^2\left(\frac{1}{4(n_1+1)}\sqrt{\frac{-p}{r}}¶\right)\right]. \tag{3.4}$$

So (3.4) gives rise to a solitary wave solutions of peak type and valley type of (1.2).

(4) Suppose that $n=2n_1+2, m=2n_1+2, m_1>n_1, m_1, n_1\in Z^+, r>0, p<0, q=0$ (see Fig. 7 (7-3)). By using similar method of (1), we obtain the parametric representation of this arch as follows:

$$¤(¶)=\pm\left[\sqrt{\frac{-p(m_1+n_1+2)}{2(n_1+1)}}\,\mathrm{sech}\left(\frac{m_1-n_1}{2(n_1+1)}\sqrt{\frac{-p}{r}}¶\right)\right]^{\frac{1}{m_1-n_1}}. \tag{3.5}$$

So (3.5) gives rise to two solitary wave solutions of peak type and valley type of (1.2).

(5) Suppose that $n=3, m=5, r>0, (p,q)\in(A_1)$ (see Fig. 2 (2-3)), corresponding to the orbit defined by $H(¤,z)=0$ to the equilibrium point $A(0,0)$, the arch curve has the algebraic equation

$$z^2=\frac{1}{12r}\left(d¤-¤^2\right)\left(¤^2+d¤+\frac{4p}{3}+d^2\right), \tag{3.6}$$

where d is the real root of the equation of $¤^2 + d¤ + \frac{4p}{3} + d^2 = 0$. Thus, by using the first equation of (1.5) and (3.6), we obtain the parametric representation of this arch as follows:

$$s(¶) = \frac{\sqrt{-\frac{q_1}{p_1}}}{cn(\Omega_1 ¶; k_1)}, \tag{3.7}$$

where $cn(x;k)$ is the Jacobin elliptic functions with the modulo k and $\Omega_1 = \frac{\sqrt{p_1 q_2 - p_2 q_1}}{(ª2 - ª1)\sqrt{12r}}$,

$k_1 = \sqrt{\frac{p_1 q_2}{p_1 q_2 - p_2 q_1}}$, $\frac{d¤}{ds} = \frac{ª2 - ª1}{(s-1)^2}$, $ª1$, $ª2$ satisfy $ª^2 + (\frac{4p}{3d} + d)ª - (\frac{2p}{3} + \frac{d^2}{2}) = 0$, $p_1 = -ª_1^2 + dª_1$,

$p_2 = ª_1^2 + dª_1 + \frac{4p}{3} + d^2$, $q_1 = -ª_2^2 + dª_2$, $q_2 = ª_2^2 + dª_2 + \frac{4p}{3} + d^2$. So (3.7) gives rise to a compacton solution of(1.2).

(6) Suppose that $n = 3, m = 5, r < 0, (p,q) \in (A_2) \cup (A_3)$ (see Fig. 2 (2-3)), corresponding to the orbit defined by $H(¤, z) = 0$ to the equilibrium point $A(0,0)$, the arch curve has the algebraic equation

$$z^2 = \frac{1}{12(-r)}(¤ - ¤_1)(¤ - ¤_2)(¤_3 - ¤)(¤_4 - ¤), \tag{3.8}$$

where $¤_1 < ¤_2 < ¤_3 < ¤_4, ¤_i(¤_i^3 + \frac{4p}{3}¤_i + \frac{8q}{5}) = 0, i = 1-4$. Thus, by using the first equation of(1.5) and (3.8), we obtain the parametric representation of this arch as follows:

$$¤(¶) = \frac{(¤_4 - ¤_2)¤_3 - ¤_4(¤_3 - ¤_2)sn^2(\Omega_2 ¶; k_2)}{(¤_4 - ¤_2) - (¤_3 - ¤_2)sn^2(\Omega_2 ¶; k_2)}, \tag{3.9}$$

where $sn(x;k)$ is the Jacobin elliptic functions with the modulo k and $\Omega_2 = \frac{1}{4}\sqrt{\frac{(¤_4 - ¤_2)(¤_3 - ¤_1)}{-3r}}$, $k_2 = \sqrt{\frac{(¤_3 - ¤_2)(¤_4 - ¤_1)}{(¤_4 - ¤_2)(¤_3 - ¤_1)}}$. So (3.9) gives rise to a compacton solution of (1.2).

(7) Suppose that $n = 4, m = 6, r > 0, (p,q) \in (B_3) \cup (B_4)$ (see Fig. 3 (3-2)), corresponding to the orbit defined by $H(¤, z) = 0$ to the equilibrium point $S_{\pm}\left(0, \sqrt{\frac{-q}{12r}}\right)$, the arch curve has the algebraic equation

$$z^2 = \frac{1}{20r}\left(\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}} - \frac{5}{8}p - ¤^2\right)\left(\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}} + \frac{5}{8}p + ¤^2\right). \tag{3.10}$$

Thus, by using the first equation of (1.5) and (3.10), we obtain the parametric representation of this arch as follows:

$$\text{¤}(\P) = \pm\left(\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}} - \frac{5}{8}p\right)^{\frac{1}{2}} cn(\Omega_3 \P; k_3), \quad \left|\P \le \frac{2K(k_3)}{\Omega_3}\right|, \tag{3.11}$$

where $\Omega_3 = \sqrt{\frac{1}{10r}\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}$, $k_3 = \sqrt{\frac{\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}} - \frac{5}{8}p}{2\sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}}$, $K(k)$ is the first kind of complete elliptic integral. So (3.11) gives rise to two periodic cusp wave solutions of peak type and valley type of of (1.2).

(8) Suppose that $n = 4, m = 6, r < 0, (p,q) \in (B_2)$ (see Fig. 3 (3-4)). By using similar method of (7), we obtain the parametric representation of this arch as follows:

$$\text{¤}(\P) = \pm\sqrt{-\frac{5}{8}p - \sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}\, sn(\Omega_4 \P; k_4), \quad \left|\P \le \frac{2K(k_4)}{\Omega_4}\right|, \tag{3.12}$$

where $\Omega_4 = \frac{1}{2}\sqrt{\frac{-\frac{5}{8}p + \sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}{5(-r)}}$, $k_4 = \sqrt{\frac{-\frac{5}{8}p - \sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}{-\frac{5}{8}p + \sqrt{\frac{25}{64}p^2 - \frac{5q}{3}}}}$. So (3.12) gives rise to two periodic cusp wave solutions of peak type and valley type of (1.2).

(9) Suppose that $n = 3, m = 4, r > 0, (p,q) \in (C_1)$ (see Fig. 4 (4-1)), corresponding to the orbit defined by $H(\text{¤}, z) = 0$ to the equilibrium point $A(0,0)$, the arch curve has the algebraic equation

$$z^2 = \frac{2}{21r}(0 - \text{¤})\left(\text{¤} + \frac{7}{12}p - \frac{1}{2}\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}\right)\left(\text{¤} + \frac{7}{12}p + \frac{1}{2}\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}\right). \tag{3.13}$$

Thus, by using the first equation of (1.5) and (3.13), we obtain the parametric representation of this arch as follows:

$$\text{¤}(\P) = \frac{\left[49p^2 - 36\left(\frac{49}{36}p^2 - \frac{28}{5}q\right)\right]\left(1 - sn^2(\Omega_5 \P; k_5)\right)}{12\left[\left(7p - 6\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}\right)sn^2(\Omega_5 \P; k_5) - \left(7p + 6\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}\right)\right]}, \tag{3.14}$$

where $\Omega_5 = \frac{1}{6}\sqrt{\frac{7p + 6\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}}{14r}}$, $k_5 = \sqrt{\frac{\frac{7}{12}p - \frac{1}{2}\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}}{\frac{7}{12}p + \frac{1}{2}\sqrt{\frac{49}{36}p^2 - \frac{28}{5}q}}}$ .So (3.14) gives rise to a compacton solution of (1.2).

(10) Suppose that $n=3, m=4, r>0, (p,q)\in(C_4)$ (see Fig. 4 (4-3)). By using similar method of (9), we obtain the parametric representation of this arch as follows:

$$¤(¶)=\frac{\frac{7}{5}qsn^2(\Omega_6¶;k_6)}{\left(-\frac{7}{12}p+\frac{1}{2}\sqrt{\frac{49}{36}p^2-\frac{28}{5}q}\right)sn^2(\Omega_6¶;k_6)-\sqrt{\frac{49}{36}p^2-\frac{28}{5}q}}, \tag{3.15}$$

where $\Omega_6=\frac{1}{2}\sqrt{\frac{2}{21r}\sqrt{\frac{49}{36}p^2-\frac{28}{5}q}}$, $k_6=\sqrt{\frac{-\frac{7}{12}p+\frac{1}{2}\sqrt{\frac{49}{36}p^2-\frac{28}{5}q}}{\sqrt{\frac{49}{36}p^2-\frac{28}{5}q}}}$.

So (3.15) gives rise to a compacton solution of (1.2).

(11) Suppose that $n=4, m=5, r>0, (p,q)\in(D_3)$ (see Fig. 5 (5-2)), corresponding to the orbit defined by $H(¤,z)=0$ to the equilibrium point $S_\pm$, the arch curve has the algebraic equation

$$z^2=\frac{1}{18r}\left(-¤^3-\frac{9p}{8}¤^2-\frac{3}{2}q\right). \tag{3.16}$$

Thus, by using the first equation of (1.5) and (3.16), we obtain the parametric representation of this arch as follows:

$$¤(¶)=-\frac{3p}{8}+\wp\left(\frac{¶}{\sqrt{72r}},g_2,g_3\right),\quad |¶|\le\frac{\sqrt{72r}K\left(\sqrt{\frac{A+b_1-l}{2A}}\right)}{\sqrt{A}}, \tag{3.17}$$

where $g_2=\frac{27}{16}p^2$, $g_3=\frac{1}{2}p^3+6q$, $A^2=(b_1-l)^2+a_1^2$, $b_1=-\frac{l}{2}$, $l=\frac{B}{8}+\frac{9p^2}{8B}$, $a_1^2=-\frac{3\left(\frac{B}{8}-\frac{9p^2}{8B}\right)}{16}$, $B=\left(32p^3+384q+\sqrt{295p^6+24576p^3q+147456q^2}\right)^{\frac{1}{3}}$ .So (3.17) gives rise to a periodic cusp wave solutions of peak type of (1.2).

(12) Suppose that $n=4, m=5, r>0, (p,q)\in(D_4)$ (see Fig. 5 (5-3)), corresponding to the orbit defined by $H(¤,z)=0$ to the equilibrium point $S_\pm$, the arch curve has the algebraic equation

$$z^2=\frac{1}{18r}\left(-¤^3-\frac{9p}{8}¤^2-\frac{3}{2}q\right)=-\frac{1}{18r}(¤-¤_3)(¤-¤_2)(¤-¤_1), \tag{3.18}$$

where $¤_3>¤_2>¤_1, ¤_i^3+\frac{9p}{8}¤_i+\frac{3}{2}q=0, i=1-3$ .Thus, by using the first equation of (1.5) and (3.18), we obtain the parametric representation of this arch as follows:

$$¤(¶) = ¤_2 + (¤_3 - ¤_2)\, cn^2(\Omega_7 ¶; k_7), \quad |¶| \le \frac{K(k_7)}{\Omega_7}, \tag{3.19}$$

where $\Omega_7 = \frac{1}{6}\sqrt{\frac{¤_3 - ¤_1}{2r}}, k_7 = \sqrt{\frac{¤_3 - ¤_2}{¤_3 - ¤_1}}$ .So (3.19) gives rise to two periodic cusp wave solutions of peak type and of valley type of (1.2).

4 Conclusion

In this paper, we have considered all solitary wave, periodic cusp wave and compactons for the KP-like K(m,n) system (1.2) in its parameter space, by using the method of dynamical systems. We obtain some parametric representations for solitary wave, periodic cusp wave and compactons of (1.1) in different parameter regions of the parameter space.

Competing interests

Authors have declared that no competing interests exist.

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